Psychological Review
1994, Vol. 101, No. 4, 734–739

0033-295X/94/$3.00

Why Models Rather Than Rules Give a Better Account of Propositional Reasoning: A Reply to Bonatti and to O'Brien, Braine, and Yang

P. N. Johnson-Laird, Ruth M. J. Byrne, and Walter Schaeken

O'Brien, Braine, and Yang argue that the mental model theory of propositional reasoning is easy to refute, and they report 3 experiments that they believe falsify the theory. In contrast, Bonatti argues that the model theory is too flexible to be falsified. We show that O'Brien et al.'s experiments do not refute the model theory and that Bonatti's claims are ill founded. Formal rule theories of propositional reasoning have 3 major weaknesses in comparison with the model theory: (a) They have no decision procedure; (b) they lack predictive power, providing no account of several robust phenomena (e.g., erroneous conclusions tend to be consistent with the premises); and (c) as a class of theories, they are difficult to refute experimentally.

For about 150 years, theorists have argued that the mind is equipped with formal rules of inference that enable it to make deductions based on "propositional" connectives such as *and, or,* and *if* (e.g., see Boole, 1854; Inhelder & Piaget, 1958; Osherson, 1974–1976; Pollock, 1989; Rips, 1994). O'Brien, Braine, and Yang (1994) are distinguished proponents of such a *rule* theory, to which Bonatti (1994) also adheres. Recently, a different sort of theory of reasoning—both deductive and inductive—has postulated that it is a semantic process in which reasoners envisage the situations described by the premises. They build *models* of these situations, mental representations that correspond in structure to the situations that they represent. Along with our colleagues, we have developed a model theory of this sort, first for syllogisms, then for spatial and other relations, including multiply quantified ones, and most recently for propositional reasoning (Johnson-Laird, Byrne, & Schaeken, 1992). The criticisms of rule theorists have helped us to strengthen the theory and its description, and we are grateful to both Bonatti and O'Brien et al. for their critiques. Our goal in this reply is to make progress toward resolving the controversy. We begin by considering what, in principle, would refute the two sorts of theories. We then explain why O'Brien et al.'s experiments do not overturn the model theory and why Bonatti's criticisms miss their target. Finally, we assess the current status of the controversy.

P. N. Johnson-Laird, Department of Psychology, Princeton University; Ruth M. J. Byrne, Department of Psychology, University of Ireland, Dublin, Ireland; Walter Schaeken, Department of Psychology, University of Leuven, Leuven, Belgium.

This work was carried out with support from the James S. McDonnell Foundation and from the National Fund for Scientific Research of Belgium.

We are grateful to Martin Braine for his help in elucidating his theory and, as ever, for conducting the controversy in an exemplary way. We thank Danny Kahneman for a useful expository suggestion.

Correspondence concerning this article should be addressed to P. N. Johnson-Laird, Department of Psychology, Princeton University, Green Hall, Princeton, New Jersey 08544. Electronic mail may be sent to phil@clarity.princeton.edu.

The Controversy Between Rule and Model Theories

According to rule theories, deduction consists of extracting logical forms of assertions and matching them to rules to derive a conclusion in a chain of deductions akin to a logical proof. Consider, for example, the following deduction:

> Either there is a short in the circuit or else the battery is dead.
> Actually, the battery is *not* dead.
> Therefore, there is a short in the circuit.

Braine's (1990) theory postulates that the first premise is analyzed as having the logical form (s v d) & not (s & d), where s stands for "there is a short in the circuit," d stands for "the battery is dead," v stands for disjunction, and & stands for conjunction. The categorical premise has the following form: not d. The following chain of deductions then ensues:

> ∴ s v d (using a rule for conjunction of the form P & Q ∴ P)
> ∴ s (using a rule for disjunction of the form P v Q, not Q ∴ P)

Note that the inference should be easier with an inclusive disjunction, represented as s v d, which obviates the need for the first step in the preceding derivation. We return to this point later.

According to the model theory, reasoners construct models of the premises and formulate a conclusion based on them. If the conclusion holds in all of the models of the premises, it is necessary (i.e., deductively "valid"); if it holds in most of the models, it is probable; and if it holds in at least some models, it is possible (consistent with the premises). In the earlier deduction, the first premise (Either there is a short in the circuit or else the battery is dead) calls for models of two alternative possibilities:

s
    d

Each line denotes a model of a separate possibility (s denotes a model of the short in the circuit, and d denotes a model of the dead battery). These models will themselves have a structure corresponding to their respective situations. The premise "The battery is *not* dead" eliminates the second model, so reasoners can conclude validly that there is a short in the circuit.

The gulf between formal and semantic methods has been established by logicians (e.g., see Quine, 1974), and the two sorts of psychological theories are so different that it ought to be possible for experiments to distinguish between them. In fact, however, such determinations are not easy in the domain of propositional reasoning. One problem is the variety of rule theories. Another is that rule theorists may legitimately modify the sorts of rules that they postulate and the strategies that deploy them; for example, Rips (1994) has, in this way, made a significant advance on his earlier system. Likewise, model theorists may legitimately modify the models that they postulate and the principles for combining them. Hence, if the controversy is to be resolved, empirical results must contravene a theory's fundamental principles.

What would contravene the fundamental principles of the class of formal rule theories? It is hard to say. The theories predict that the difficulty of a deduction depends on the length of its derivation and the accessibility (or ease of manipulation) of each rule used in the derivation. But because theorists may legitimately change the rules and the strategy for deploying them, it is hard to find a decisive test. Perhaps an experiment can test only a specific rule theory. Unfortunately, as Bonatti (1994) concedes, there are no rule theories for many domains of deduction.

What would contravene the fundamental principles of the model theory? In fact, to echo O'Brien et al.'s (1994) title, the theory is simple to refute in principle. It applies to all domains of deduction, and it makes two general predictions about them. The first prediction is easy to test, because it does not call for any account of the specific models for a domain. According to this prediction, erroneous conclusions should be consistent with the premises rather than inconsistent with them, because reasoners will err by basing their conclusions on only some of the models of the premises. They will accordingly draw a conclusion that is possibly true rather than necessarily true. To test the prediction, one examines whether erroneous conclusions are consistent with the premises more often than not. This prediction provides an alternative explanation for the "atmosphere" effect in syllogistic reasoning, and it has been corroborated in all of the main domains of deduction, including propositional reasoning (e.g., see Figure 1 of Johnson-Laird et al., 1992).

The second prediction is that the greater the number of models that have to be constructed to make an inference, the harder the task will be; reasoners will take longer to respond and be more likely to err. In some domains, such as spatial or temporal reasoning, the number of models is easy to assess. In other domains, such as syllogisms, the prediction requires a detailed account of the models constructed from premises. Propositional reasoning stands somewhere between the two. Obviously, a conjunction (*a* and *b*) calls for one model that satisfies both conjuncts, an exclusive disjunction (*a* or else *b*) calls for two models, and an inclusive disjunction (*a* or *b*) calls for three models (one for *a*, one for *b*, and one for their joint occurrence). It would be hard to imagine a version of the model theory that violated these numbers, which are borne out by empirical evidence (Johnson-Laird et al., 1992).

What complicates matters are conditionals. A conditional, such as "If there is an *A* on the board, then there is a *2* on the board," means that one possible state of affairs is that there is an *A* and a *2* on the board. But there must be at least one other possibility or else the conditional would be equivalent to the conjunction: *A* and *2*. Another possibility is, indeed, that there is not an *A* on the board. Is there then a *2* on the board or not? The model theory assumes that individuals do not make the possibilities explicit if they do not matter. Hence, the conditional has the initial models

A *2*
. . .

where the ellipsis represents the other possibilities. However, reasoners need to make a mental footnote that *A*s cannot occur in the alternative possibilities because they are exhaustively represented in the explicit model. The model theory postulates that reasoners perform at different levels of expertise depending on whether they omit or forget the footnote, use it to guide the construction of new models, or make its content explicit in new models.

If more models imply more work for reasoners, it is important—as Bonatti (1994) correctly points out—to be clear as to how to count them. In the case of syllogisms, it is simple. Each premise calls for one model containing two sets of elements, and all that varies is the number of models when the two models are combined. In the case of propositional reasoning, it is harder. The number of propositions represented in a model can vary, different premises can call for different numbers of models, and the results of combining them can differ. An ideal measure of the load on working memory should be sensitive to all of these variables. No single measure is likely to do an optimal job in all cases. When a set of inferences is highly varied, the best policy is probably to count the total number of models constructed in the course of arriving at the conclusion. O'Brien et al. (1994) quote us to the effect that any inference calling for three or more explicit models should be almost impossibly difficult. They overlook the context of our claim (Johnson-Laird et al., 1992, p. 434): The difficult problems are those for which the correct conclusions correspond to three or more models. If three models per se were difficult, then subjects would have great difficulty in understanding an inclusive disjunction. Moreover, the subjects in our experiments were from the public at large. We now know that students at Princeton can cope with rather more models (Bauer & Johnson-Laird, 1993).

In summary, the model theory is easier to refute than the rule theories. Has it, in fact, been refuted? The answer is to be found in the next two sections.

Do O'Brien et al.'s (1994) Results Refute the Model Theory?

O'Brien et al. (1994) report three experimental results, and the third of them establishes an important new phenomenon in deductive reasoning. They interpret each of their results as corroborating the rule theory but as contrary to the model theory. But they base their predictions from the model theory only on its most rudimentary level of performance, in which footnotes have been omitted or forgotten (see the previous section), and, like Bonatti (1994), they almost entirely overlook performance with footnotes. This oversight, as we show later, vitiates their criticisms.

Their first result is that subjects do not commit certain fallacies that, according to O'Brien et al. (1994), the model theory allows. This defect, they state, is sufficiently serious that it alone impeaches the theory. They claim that the only way that the model theory can satisfactorily block the fallacies is by fleshing out the models of conditional premises in a wholly explicit way. The claim is true if one considers the two computer programs described in Johnson-Laird et al. (1992); one implements the most rudimentary reasoning (with no footnotes) and the other is an Artificial Intelligence program that reasons with completely explicit models. But the claim overlooks the role of footnotes at an intermediate level of performance. Consider a fallacy of the same form as O'Brien et al.'s Problem 4:

If *A* and *B* then *C*
A
∴ *C*

At one level up from the most rudimentary performance, the conditional premise is represented as follows (e.g., see Byrne & Johnson-Laird, 1990, p. 140):

[*A* *B*] *C*
. . .

The brackets are a footnote indicating that the *conjunction* of *A* and *B* cannot occur among the alternative possibilities represented by the implicit model. There is nothing to prevent *A* alone as an alternative possibility, however, so the result of combining these models with the model for the categorical premise, *A*, is

A *B* *C*
A

C is not a valid conclusion because it does not hold in the second of these models. Hence, the fallacy is blocked even though the models are not fleshed out completely. The other fallacies used by O'Brien et al. are also blocked in this way, although, in fairness to them, we did not describe this mechanism in Johnson-Laird et al. (1992).

The second result can be illustrated by the following deduction from O'Brien et al. (1994):

If *S* or *X* or *B* or *C* or *K* or *R* or *N* or *L* or *D* or *F* then not both *I* and *Q*
X
∴ not both *I* and *Q*

Most subjects accepted the conclusion as valid. The first premise supports many possible models, so O'Brien et al. take the ease of this and other similar deductions to count against the prediction that multiple models should yield difficult deductions. Bonatti (1994) supports a similar claim with a similar example. What these claims overlook, however, is that people do not blindly build models just for the sake of doing so. In the preceding example, there is no need to construct all of the possible models of the conditional. Indeed, subjects may not even bother to read the full antecedent. The second premise, *X*, establishes the truth of the conditional's antecedent regardless of the number and nature of its other disjunctive components; thus, the other disjuncts can be represented as implicit alternatives. Similarly, there is no need to build detailed models of the conditional's consequent. Reasoners can easily see that it matches the putative conclusion. Hence, reasoners have only to make a deduction of the form

X
If *X* or . . . then *B*
∴ *B*

where *B* = "not both *I* and *Q*." This deduction calls for an entirely manageable number of models. We used precisely such matches in the semantic account of propositional reasoning that was a precursor to the model theory (see Johnson-Laird, 1983, p. 46), although, in fairness to O'Brien et al., the algorithm was not intended as a fully realistic account of mental processes.

The third result is of considerable intrinsic interest quite apart from the controversy. It concerns the order in which reasoners make a chain of deductions. For example, given premises of the form,

N or *P*,
Not *N*,
If *P* then *H*,
If *H* then *Z*, and
Not both *Z* and *S*,

O'Brien et al.'s (1994) subjects tended to draw the following sequence of conclusions: *P*, *H*, *Z*, Not *S*. And they did so even when the premises were presented in exactly the opposite order. O'Brien et al. argue that the order of conclusions depends on the order of application of formal rules and that the model theory provides no reason to expect the same order of conclusions across the two problems. They also argue that the very idea that subjects might draw a sequence of intermediate conclusions is to be expected from the rule theory but not from the model theory.

It is true that the model theory has not hitherto been applied to the order of intermediate conclusions. Yet, the underlying principles of the theory (Johnson-Laird, 1983) yield a natural prediction: If possible, reasoners will tend to start with a premise that is maximally semantically informative (e.g., a categorical assertion), and they will tend to work in an order that maintains co-reference, drawing, when possible, informative intermediate conclusions. It makes sense to base an inference on a firm footing (i.e., one model), and the predicted order also enables reasoners to minimize the number of models of alternative possibilities that they have to construct, an idea that is implemented in a program using models for temporal reasoning (Schaeken, Johnson-Laird, & d'Ydewalle, 1993). Consider, for instance, the following "double disjunction":

A or *B*, or both
not-*B* or *C*, or both
not-*A*

If reasoners interpret the premises in the order in which they are stated, the first two premises yield four models. But if they start with the categorical premise, not-*A*, they can combine it with the first premise to yield a single model, which in turn yields a single model when it is combined with the second premise. There is a "figural effect" in drawing conclusions (i.e., a conclusion tends to be stated in the same order as its elements entered working memory). O'Brien et al. (1994) mention this phenomenon as though it were incompatible with choosing an optimal order. In fact, the discovery of the figural effect occurred precisely because subjects do not always interpret premises in their order of mention (see Johnson-Laird, 1983, p. 105). In summary, none of O'Brien et al.'s experimental results impugn the model theory. It remains to be seen whether the model theory's account of the order of intermediate deductions diverges from the predictions of the rule theory.

Is Bonatti's (1994) Article a Reliable Guide to Either Theory?

Bonatti's (1994) article presents no new data and no new theoretical hypotheses. It argues that the model theory is "ill-defined," "self-refuting," "flawed in a way which is difficult to overcome," and "can be made consistent with almost everything one wants," whereas "the only theory of mental logic that . . . can approach a psychologically real model of human reasoning is the one developed by Braine and his collaborators" (1994, p. 725). The article makes some good points, such as the importance of the method of counting models (see earlier discussion). Unfortunately, its adversarial attitude leads it into error, and it is not a reliable guide to the model theory or to Braine's (1990) theory.

Bonatti (1994) argues that a major flaw in the model theory is that it does not represent the truth conditions of sentences and therefore cannot represent contradictions. We can do no better than to repeat what we wrote in refereeing the article:

> This brief argument [of Bonatti's] is based on two errors. First, the model theory *does* represent truth conditions (in the initial linguistic representation that is used to construct models, see Johnson-Laird and Byrne, 1991, Ch. 9, for an account of how parsing yields these representations). Second, when the truth conditions of a contradiction are evaluated, they yield the null model. In short, the model theory has no difficulty in coping with self-contradictions—either alone or as constituents of conditionals or other sort of assertions.

At the most rudimentary level of performance, according to the model theory, reasoners treat conditionals as conjunctionlike; that is, implicit models are, in effect, forgotten because they yield the null model whenever they are combined with an explicit model. Bonatti points out some of the illogicalities that then occur. Should we therefore abandon this level of performance as part of the theory? Probably not. Several investigators have reported that children interpret conditionals in a conjunctionlike way. For example, O'Brien (1987) wrote:

> Braine, O'Brien, and Connell found that young children persist in making conjunction-like responses even when provided with evidence to the contrary. . . . Most of the 7 and 9 year olds, and even some adults, judged *if there is an apple in the box then there is a horse* as true even though there was also a box with both an apple and a dog; they responded as though the sentence means *There is a box with both an apple and a horse.* (pp. 74–75)

O'Brien argued against the idea that children interpret conditionals as equivalent to conjunctions. Yet, in an unpublished study, Johnson-Laird and Barres observed that adults do interpret conditionals as conjunctions in certain compound assertions. Thus, they consider assertions of the form "If *A* then 2, or if *B* then 3" to be true in just those cases that they consider assertions of the form "*A* and 2, or *B* and 3" to be true. This prediction was an unexpected consequence of the model theory. Can it be derived, we wonder, from Braine's theory?

Bonatti's most serious criticism is that the model theory is "so flexible that it can be made consistent with almost everything one wants" (1994, p. 727). On this point, as we have already shown, O'Brien et al. are right: The theory is simple to refute in principle. Bonatti goes on to argue that our AI algorithm, which reasons with explicit models, is psychologically useless. Readers might infer that he is making a claim contrary to our views. In fact, as we wrote, the point of the algorithm was to make all and only the valid deductions in the propositional calculus and to develop a procedure for drawing parsimonious conclusions that can outperform the standard "prime implicant" method.

In his defense of Braine's (1990) rule theory, Bonatti (1994) tries to argue that it makes the same predictions as the model theory. He considers three phenomena demonstrated by Johnson-Laird et al. (1992).

1. A deduction of the *modus ponens* form

If *p* then *q*
p
∴ *q*

is easier than a disjunctive deduction of the form

p or else *q* (not both)
p
∴ not *q*

The model theory makes this prediction because the conditional calls for only one explicit model, whereas the exclusive disjunction calls for two. Bonatti (1994) argues that Braine's (1990) theory makes "roughly the same prediction" (p. 729) because *modus ponens* calls for only a single rule, whereas the disjunctive deduction calls for more than one rule. This claim is true, but other rule theories allow the disjunctive deduction to be made directly by a single rule. The case is a good example of how difficult it is to refute the class of formal rule theories in general.

2. The model theory predicts that the difference between *modus ponens* and *modus tollens* (if *p* then *q*; not *q*; therefore not *p*) should be reduced in the case of a biconditional as opposed to a one-way conditional, because fleshing out the former yields only two explicit models, whereas fleshing out the latter yields three explicit models. Although Braine's (1990) theory can explain the difference between *modus ponens* and *modus tollens,* Bonatti (1994) concedes that it cannot predict the interaction.

3. The model theory predicts that double disjunctions of the form

A or *B*
B or *C*
What follows?

should be easier with exclusive disjunctions than with inclusive disjunctions and that they should be easier with a proposition in common to both premises (as B is here) than with a proposition in one premise that is contrary to a proposition in the other premise. Bonatti (1994) asserts the following in regard to Braine's (1990) theory: "By application of [its] direct reasoning routines alone both exclusive and inclusive premises lead to no conclusion; hence, 'nothing follows' conclusions are predicted as frequent responses" (p. 730). And he reports (wrongly) that our data show that, in all cases, the most frequent answer was "nothing follows" (p. 730). In fact, erroneous conclusions based on one model of the premises were more frequent for both sorts of exclusive disjunction (see Figure 1 of Johnson-Laird et al., 1992). Moreover, in the main double disjunction experiment reported by Bauer and Johnson-Laird (1993), the subjects never responded "nothing follows." If such a response is really predicted to be the most frequent by Braine's theory, then the theory is refuted by this result.

Why should exclusive disjunctions be easier than inclusive disjunctions? The model theory has a simple answer: fewer models. Thus, the model theory predicts that an inference of the form

> There is a short in the circuit or the battery is dead.
> The battery is *not* dead.
> Therefore, there is a short in the circuit.

should be easier with an exclusive disjunction than with an inclusive disjunction. Braine's (1990) theory makes the opposite prediction because the exclusive disjunction calls for an extra step (see the earlier section on the controversy between the theories). The evidence supports the model theory (see Evans, Newstead, & Byrne, 1993, chap. 5). Even if a rule theory postulates a separate rule of inference for exclusive disjunctions of the form

> *p* or else *q*
> not *p*
> ∴ *q*

it does not explain why exclusive disjunctions are easier than inclusive disjunctions. It can only assume that the exclusive rule is easier to use than the inclusive rule.

The biggest weakness in Bonatti's (1994) discussion of double disjunctions is that he says nothing whatsoever about two striking findings. First, most erroneous conclusions were consistent with the premises. Second, diagrams that make the alternative possibilities more explicit yield a radical improvement in performance: Subjects are both faster and draw more valid conclusions (Bauer & Johnson-Laird, 1993). Both phenomena are predicted by the model theory; neither are predicted by formal rule theories.

Finally, to his credit, Bonatti (1994) does identify a major problem with formal rule theories. Having dismissed a certain strategy for double disjunctions, he writes:

> Other, more difficult, strategies must be deployed, and even so, looking for a conclusion is more a blind search than a reasoning process. In such a case, then, no derivation can be directly reached if a conclusion does not indicate which assumptions to make or where to look for a derivation: People just do not know how to continue the story. (1994, p. 730)

In Bauer and Johnson-Laird's (1993) experiment, the subjects did know how to continue the story: They performed well. But Bonatti is right that the quest for a formal derivation is a blind search.

Conclusions

To readers who are innocent bystanders, the controversy between formal rule theories and the mental model theory may seem to be just another standoff in the long history of such episodes in psychology. All parties to the dispute, however, concur on several important matters: Deductive ability is a central component of human mentality; theories should be modeled in algorithms; and psychological experiments should be able to resolve the controversy. So what is the controversy's current status? We answer both at the level of empirical observations and at a broader theoretical level.

The experimental results of O'Brien et al. (1994) do not overturn the model theory. The theory allows the fallacies in their experiments only at its most rudimentary level of performance; it is not refuted by simple inferences that call for many models if subjects can easily avoid having to construct them; and it can make predictions about the optimal order of interpreting premises. Conversely, at least two phenomena corroborate the model theory but run counter to Braine's (1990) formal rule theory: Direct inferences are easier with exclusive disjunctions than with inclusive disjunctions, and *if* is interpreted in certain contexts as equivalent to *and.* If the results had been otherwise, then the model theory would have been refuted (with due respect to Bonatti, 1994). To the best of our knowledge, there are no results that corroborate formal rule theories but run counter to the model theory.

There are crucial differences between the two approaches at a broader theoretical level. First, theories based on formal rules lack a decision procedure. To paraphrase Quine (1974, p. 75), they afford no general way of reaching a verdict of invalidity; failure to discover a proof can mean either invalidity or mere bad luck (see also Barwise, 1993). In contrast, the model theory provides a decision procedure for propositional reasoning; a conclusion is valid if and only if it is true of all of the models of the premises.

Second, rule theories have a smaller scope than the model theory. They apply to fewer domains of reasoning, and, even within propositional reasoning, they provide no account of several robust phenomena. They do not predict that erroneous conclusions will tend to be consistent with the premises; they do not predict that diagrams making alternative possibilities more explicit will improve reasoning; and, as yet, they give no account of modal inferences such as the following:

> There is a short in the circuit or the battery is dead.
> ∴ It is possible that there is a short in the circuit *and* the battery is dead.

In all of these cases, they run the risk of "not even being false."

Third, it is hard for empirical results to contravene the fundamental principles of the class of formal rule theories. Their refutation is a Herculean task: Bring down one theory and another can spring up to take its place. In contrast, the model theory remains easy to refute in principle. Do we imply that the model theory is a paragon? Of course not. There are phenomena for which it offers no satisfactory account, such as the vagaries in the frequencies with which various fallacies occur (see Evans et al., 1993). Likewise, there are aspects of the theory that have yet to be implemented in computer models, such as the predictions about the optimal order of interpreting premises so nicely revealed in O'Brien et al.'s (1990) experiment. In our view, the ultimate resolution of the controversy is likely to depend on rule theorists extending their theories to make predictions about all of the phenomena explained by the model theory and on the development of crucial experiments that distinguish between the two approaches.

References

Barwise, J. (1993). Everyday reasoning and logical inference. *Behavioral and Brain Sciences, 16,* 337–338.

Bauer, M. I., & Johnson-Laird, P. N. (1993). How diagrams can improve reasoning. *Psychological Science, 4,* 372–378.

Bonatti, L. (1994). Propositional reasoning by model? *Psychological Review, 101,* 725–733.

Boole, G. (1854). *An investigation of the laws of thought on which are founded the mathematical theories of logic and probabilities.* New York: Macmillan.

Braine, M. D. S. (1990). The "natural logic" approach to reasoning. In W. F. Overton (Ed.), *Reasoning, necessity, and logic: Developmental perspectives.* Hillsdale, NJ: Erlbaum.

Byrne, R. M. J., & Johnson-Laird, P. N. (1990). Models and deductive reasoning. In K. J. Gilhooly, M. T. G. Keane, R. H. Logie, & G. Erdos (Eds.), *Lines of thinking* (Vol. 1). New York: Wiley.

Evans, J. S. B. T., Newstead, S. E., & Byrne, R. M. J. (1993). *Human reasoning: The psychology of deduction.* Hillsdale, NJ: Erlbaum.

Inhelder, B., & Piaget, J. (1958). *The growth of logical thinking from childhood to adolescence.* London: Routledge & Kegan Paul.

Johnson-Laird, P. N. (1983). *Mental models.* Cambridge, England: Cambridge University Press.

Johnson-Laird, P. N., & Byrne, R. M. J. (1991). *Deduction.* Hillsdale, NJ: Erlbaum.

Johnson-Laird, P. N., Byrne, R. M. J., & Schaeken, W. (1992). Propositional reasoning by model. *Psychological Review, 99,* 418–439.

O'Brien, D. P. (1987). The development of conditional reasoning: An iffy proposition. *Advances in Child Development, 20,* 61–90.

O'Brien, D. P., Braine, M. D. S., & Yang, Y. (1994). Propositional reasoning by mental models? Simple to refute in principle and in practice. *Psychological Review, 101,* 711–724.

Osherson, D. N. (1974–1976). *Logical abilities in children* (Vols. 1–4). Hillsdale, NJ: Erlbaum.

Pollock, J. (1989). *How to build a person: A prolegomenon.* Cambridge, MA: Bradford Books.

Quine, W. V. O. (1974). *Methods of logic* (3rd ed.). London: Routledge & Kegan Paul.

Rips, L. J. (1994). *The psychology of proof.* Cambridge, MA: MIT Press.

Schaeken, W., Johnson-Laird, P. N., & d'Ydewalle, G. (1993). *Mental models and temporal reasoning.* Manuscript submitted for publication.

Received March 14, 1994
Revision received May 16, 1994
Accepted May 17, 1994 ■