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The Effect of an Irrelevant Premise on Temporal and Spatial Reasoning

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Zum Einfluß irrelevanter Prämissen auf das zeitliche und räumliche Schließen

Zusammenfassung. Gemäß der Theorie mentaler Modelle ziehen Individuen Schlußfolgerungen, indem sie Modelle der Situation konstruieren, die in den Prämissen beschrieben wird. Eine Aufgabe ist umso schwieriger, je mehr Modelle dabei konstruiert werden müssen. Byrne und Johnson-Laird (1989) bestätigten diese Vorhersage für räumliche Probleme, und Schaeken, Johnson-Laird und d'Ydewalle (1996a, 1996b) für zeitliche. Allerdings existiert in all diesen Arbeiten das folgende Problem: Alle Mehr-Modell-Aufgaben und einige der Ein-Modell-Aufgaben enthielten eine irrelevante Prämisse. Wie Rips (1994) anmerkte, ist es möglich, daß eine irrelevante Prämisse die Suche nach einer Ableitung behindert. In dieser Arbeit wird über ein Experiment berichtet, in dem explizit die Präsentation einer solchen irrelevanten Prämisse sowohl in Ein-Modell- als auch in Mehr-Modell-Fällen variiert wurde. Die Ergebnisse stützen die Vorhersagen der Theorie mentaler Modelle.

Summary. The mental model theory of reasoning postulates that individuals reason by constructing models of the situation described by premises. The more models reasoners have to build, the harder a problem will be. Byrne and Johnson-Laird (1989) confirmed this prediction with spatial problems and Schaeken et al. (1996a, b) with temporal ones. There is, however, a problem with these studies. All the multiple-model problems and some of the one-model problems contained an irrelevant premise. As Rips (1994) argues, it is possible that an irrelevant premise would complicate the search for a derivation. The present paper reports an experiment which explicitly manipulated the presence of such an irrelevant premise in both one-model and multiple-model problems. The results corroborate the predictions of the mental model theory.

1 Introduction

The psychology of reasoning commonly holds that reasoning depends on mental rules of inference akin to those of a logical calculus (e.g., Braine et al. 1984; Macnamara 1986; Osherson 1975; Pollock 1989; Rips 1983, 1994). These theories assume that the degree of difficulty of a problem can be measured by the number of rules applied. There is, however, an alternative theory of reasoning: the theory of mental models (Johnson-Laird 1983; Johnson-Laird & Byrne 1991). According to this theory, deductive reasoning depends on constructing a set of models based on the premises, formulating a conclusion, and ensuring that no model of the premises falsifies it. This theory states that problems are more difficult if they require more models because of the limited processing capacity of working memory.

Byrne and Johnson-Laird (1989) compared these two sorts of theories, using two-dimensional spatial reasoning problems, such as *problem 1*:

A is on the right of B.
B is on the right of C.
D is in front of C.
E is in front of B.
What is the relation between D and E?

The above problem is a one-model problem, i.e., the premises are consistent with one mental model:

C	B	A
D	E	

This model supports the conclusion:

D is on the left of E.

This problem was then compared with others, such as *problem 2*:

B is on the right of A.
C is on the left of B.
D is in front of C.
E is in front of B.
What is the relation between D and E?

The premises of this problem are consistent with at least two distinct models:

A	C	B		C	A	B
	D	E		D		E

Both models, however, support the conclusion that D is on the left of E. One-model problems were shown to be reliably easier than multiple-model problems with a valid conclusion, as the mental model theory predicts.

Rule-based theories (see Hagert 1985), however, make the opposite prediction. We will provide only a general outline of these theories here (for a more detailed account, see Byrne & Johnson-Laird 1989). In order to solve problem 1, reasoners must infer the relation between the pair of items to which D and E (the items in the question) are directly related. In order to make this inference, they must use a meaning postulate that captures the transitivity of the relations in the premises. Multiple-model problems, such as problem 2, do not require the use of such a meaning postulate. The first premise is irrelevant, and the second explicitly shows the relation between the pair of items to which D and E are related. Therefore, according to the rule-based theory of Hagert (1985), problem 2 should be easier than problem 1; however, the results did not corroborate this prediction.

Schaeken et al. (1996 a) reported similar results using temporal reasoning problems of the following sort, where A, B, C etc. refer to everyday activities, such as in *problem 3*:

A happened before B happened.
C happened after B happened.
D happened while B happened.
E happened while C happened.
What is the relation between D and E?

Schaeken et al. (1996 b) corroborated these findings in a study of temporal relations established by tense and aspect. *Problem 4* is such a problem:

John has cleaned the house.
John is taking a shower.
John is going to read the paper.
Mary always does the dishes when John cleans the house.
Mary always drinks her coffee when John reads the paper.
What is the temporal relation between Mary doing the dishes and drinking her coffee?

The above problem is an example of a one-model problem. If one changes, for instance, the second premise into a premise in the past tense, then one has an example of a multiple-model problem with a valid conclusion.

Schaeken et al. (1996 a, b) confirmed the model predictions about accuracy. They also showed that reasoners need more time to solve a multiple-model problem than to solve a one-model problem. The model theory makes an even more detailed prediction. That premise which forces reasoners to build a second model leads to the increase in time. Compare, for example, *problem 3* with the multiple-model *problem 5*:

B happened before A happened.
C happened after B happened.
D happened while B happened.
E happened while C happened.
What is the relation between D and E?

The second premise calls for the construction of alternative models in the multiple-model case, but not in the one-model case. The reading times confirmed this prediction. Hence these studies seem to support the model theory of reasoning.

Nevertheless, Rips (1994) made a serious criticism. He argued that the study carried out by Byrne and Johnson-Laird (1989) was not convincing. He raised three potential difficulties with their experiments: „First, the instructions specifically requested subjects to form spatial arrays in solving the problems. This obviously biased the subjects to use an imaginal strategy that favored the mental model predictions (or placed a strong task demand on them to respond as if they were trying to image the arrays…). Second, there is no reason at all to suppose that the final length of a derivation is the only pertinent factor … searching for a correct proof can be sidetracked by the presence of irrelevant information, such as the first premise of these problems. Third, Byrne and Johnson-Laird's method of counting models in these problems violate a policy that Johnson-Laird established elsewhere… for other types of inferences, a problem requires more than one model only if additional models rule out potential conclusions that are consistent with the initial model (Rips 1994, p. 415).

We will deal with each of these points in turn. First, the experimenter in the Byrne and Johnson-Laird (1989) study asked the participants to imagine the objects as laid out on a table top (hence there was no mention of spatial arrays). This instruction, however, may have biased the participants to adopt an otherwise alien strategy, though we doubt whether it was critical. We are also sceptical about the claim that participants yielded to a task demand to respond as if they were trying to image the arrays. How could participants respond in this way if they had never formed a spatial image? And if they were biased in this way, would it not be easier to simply form a spatial image rather than to try to simulate the process? In any case, the studies of temporal reasoning (Schaeken et al. 1996 a, b) gave no instruction about imagining arrays; they could hardly do so, because they concerned temporal events. Nevertheless, these studies showed exactly the same pattern as the studies on spatial reasoning, i. e., they support the model theory.

Second, the model theory postulates indeed that reasoners do not normally bother to construct alternative models that differ in ways that are irrelevant to the task at hand. When solving the spatial problems above, for example, they should not construct alternatives that differ merely in the relative distances apart of the objects. But consider the initial premises of the second spatial problem:

B is on the right of A.
C is on the left of B.

If a reasoner were to construct only one model:

C A B

and to overlook the other possible model:

A C B

the reasoner would go wrong in coping with the rest of the problem if it turned out not to be the multiple-model problem above, but to be the problem with no valid conclusion:

B is on the right of A.
C is on the left of B.
D is in front of C.
E is in front of A.
What is the relation between D and E?

Hence, for the multiple-model problem above, the reasoners cannot know that it is unnecessary to construct the alternative model until they have solved the problem, and by that time it is too late. The general principle that "a problem requires more than one model only if additional models rule out potential conclusions that are consistent with the initial model" remains correct. What Rips has overlooked is that participants are in no position to know that one model will suffice for those multiple-model problems with valid conclusions until they have solved the problem.

Third, the first premise is irrelevant in both the nontransitive one-model problems and the multiple-model problems: The problems can be correctly solved without reading this premise. It is possible that an irrelevant premise could complicate the search for a derivation, but there is no reason why it should be reliably harder for the multiple-model problems than for the nontransitive one-model problems. Indeed, the formal derivation is identical in both cases. Moreover, Schaeken et al.'s fifth experiment (1996 a) compared temporal one-model problems containing an irrelevant premise (i.e., nontransitive one-model problems) with one-model problems containing no irrelevant premise (i.e., transitive one-model problems). No difference was found between these two sorts of problems (in terms of accuracy and in terms of latency), so it would seem that an irrelevant premise does not lead reasoners astray. Nevertheless, a fair comparison would include multiple-model problems both with and without an irrelevant premise. Therefore, the present experiment on temporal and spatial reasoning compared one-model problems with and without an irrelevant premise and multiple-model problems with and without an irrelevant premise.

2 The Experiment

In order to build multiple-model problems without irrelevant premises, we asked the participants to imagine two kinds of cards. One kind had two cells, each of which contained a geometrical shape, as in the following cases:

O	△

The other kind had only one cell, which contained a letter:

E

In addition to the shape or the letter, a symbol could also appear in the cells: * or $.

With this kind of material, we developed one-model and multiple-model problems with and without an irrelevant premise. Consider the one-model problem with no irrelevant premise:

The square is on the left of the triangle.
The triangle is on the left of C.
* is in the same cell as the square.
$ is in the same cell as C.
What is the relation between * and $?

This problem is consistent with one model:

□*	△		C$

The model supports the conclusion that * is on the left of $. If reasoners solve this problem using rules of inference, they must infer from the first two premises the relation between the items to which * and $ are directly related.

Consider the one-model problem with an irrelevant first premise:

The square is on the left of the triangle.
The triangle is on the left of C.
* is in the same cell as the triangle.
$ is in the same cell as C.
What is the relation between * and $?

This problem is consistent with one model:

□	△*		C$

The model supports the conclusion that * is on the left of $. If reasoners solve this problem using rules of inference, they can ignore the first premise since the second premise explicitly gives the relation between the two items to which * and $ are directly related.

Consider the multiple-model problem with no irrelevant premise:

The square is on the same card as the triangle.
The triangle is on the left of C.
* is in the same cell as the square.
$ is in the same cell as C.
What is the relation between * and $?

This problem is consistent with at least two distinct models:

□*	△		C$		△	□*		C$

Both of these models, however, support the conclusion that * is on the left of $. If reasoners use rules of inference, they must infer from the first pair of premises the relation between the items (C and square) to which * and $ are directly related.

Consider the multiple-model problem with an irrelevant premise:

The square is on the same card as the triangle.
The triangle is on the left of C.
* is in the same cell as the triangle.
$ is in the same cell as C.
What is the relation between * and $?

This problem is consistent with at least two distinct models:

□	△*		C$		△*	□		C$

Both models, however, support the conclusion that * is on the left of $. If reasoners use rules of inference, they can ignore the first premise, because the second premise explicitly gives the relation between the two items (C and triangle) to which * and $ are directly related.

3 Method

3.1 Participants

We tested 280 first-year medical students, who participated voluntarily. They had no formal training in logic, and they had participated in no previous studies on temporal or spatial reasoning.

3.2 Materials

The materials for the spatial problems were described earlier. The temporal problems dealt with the temporal order of cartoons on television networks. We told the participants that the daily program of the cable network Child Television is divided into two parts. In one part, two long cartoons are broadcast. In the other, a long educational documentary is broadcast. In addition, each program can be interrupted by one advertisement.

The following is an example of a temporal problem:

The cartoon "The Mighty Mouse" was before the cartoon "The Elegant Elephant."
The cartoon "The Elegant Elephant" was before the documentary "News from Nature."
The ad about toys was during the cartoon "The Mighty Mouse."
The ad about biscuits was during the documentary "News from Nature."
What is the relation between the ad about toys and about biscuits?

The cartoon names consisted of definite descriptions (article, adjective, noun) in which the adjective and noun began with the same consonant in Flemish in order to help the participants.

Eight versions of each problem were constructed. First, four versions were constructed by systematically manipulating the spatial or temporal relations ("before" and "after") in the first two premises. In the first version, both premises contained "before," and in the second version the first premise contained "before" and the second "after." The first premise of the third version contained "after" and the second premise "before," while in the fourth version both premises contained "after." We then constructed two different variants of each of the four versions by manipulating the order of the third and fourth premises and the order of the two items in the final question.

3.3 Procedure

The participants were tested in one group. The participants received one spatial problem and one temporal problem. Half of the participants were given the spatial problem first, while the other half began with the temporal problem. Before receiving the second problem, the participants had to solve four filler items (two categorical syllogisms and two propositional problems). The spatial and temporal problems were selected randomly. Counterbalancing of the eight different versions of the problems was nearly accomplished.

Each problem, together with the instructions, was printed on a separate page in the booklet, and the participants had to write the answer under the question. The instructions explained that the participant's task was to answer a question based on the information about the arrangement of cards (in the case of the spatial problem) or the order of cartoons (in the case of the temporal problem) in the preceding statements and that the answer *must* be true given the truth of the previous statements. If the participants thought that there was no definite answer, they had to write that down as their response.

4 Results

The number of correct responses for the spatial problems were as follows:

- One-model problem with an irrelevant premise: 64 %
- One-model problem without an irrelevant premise: 66 %
- Multiple-model problem with an irrelevant premise: 59 %
- Multiple-model problem without an irrelevant premise: 40 %

As predicted by the model theory, the one-model problems were easier than the multiple-model problems (65 % vs. 49 % problems correct; binomial test, $z = 1.66$, $P < 0.05$). There was no significant difference between the two problems with or without an irrelevant premise (61 % vs. 53 % problems correct). No significant difference was found between the two one-model problems (65 % vs. 66 %) and between the two multiple-model problems (59 % vs. 40 %). The difference between the one-model problems and the multiple-model problems with no irrelevant premise was significant (66 % vs. 40 % problems correct, binomial test, $z = 1.98$, $P < 0.05$), but the difference between the one-model problems and the multiple-model problems with an irrelevant premise was not (64 % vs. 59 % problems correct).

The pattern for the temporal problems is almost identical. The number of correct responses was as follows:

- One-model problem with an irrelevant premise: 64 %
- One-model problem without an irrelevant premise: 67 %
- Multiple-model problem with an irrelevant premise: 53 %
- Multiple-model problem without an irrelevant premise: 46 %

As predicted by the model theory, the one-model problems were easier than the multiple-model problems (66 % vs. 49 % problems correct; binomial test, $z = 1.73$, $P < 0.05$). There was no significant difference between the two problems with or without an irrelevant premise (59 % vs. 56 % problems correct), neither between the two one-model problems (64 % vs. 67 %), nor between the two multiple-model problems (53 % vs. 46 %). The difference between the one-model problems and the multiple-model problems with no irrelevant premise was almost significant (67 % vs. 46 % problems correct, binomial test, $z = 1.58$, $P < 0.06$), while the difference between the one-model problems and the multiple-model problems with an irrelevant premise was not significant (64 % vs. 53 % problems correct).

5 General Discussion

The model theory currently appears to offer the most comprehensive account of spatial and temporal reasoning. Byrne and Johnson-Laird (1989) and Vandierendonck and De Vooght (1996) showed that the model theory offers the best explanation for spatial reasoning (see also Carreiras & Santamaria 1997). Furthermore, Schaeken et al. (1996 a, b) and Vandierendonck and De Vooght (1996) showed that the model theory offers the best explanation for temporal reasoning.

The present experiment focused on a criticism of Rips (1994). He argued that, in the experiments conducted by Byrne and Johnson-Laird (1989), the presence of an irrelevant premise could have influenced the reasoning process since "searching for a correct proof can be sidetracked by the presence of irrelevant information" (Rips 1994, p. 415). Schaeken et al. (1996 a) compared one-model problems with and without an irrelevant premise and found no difference between the two.

The present experiment offered a more stringent test of the hypothesis of Rips (1994), using one-model and multiple-model problems with and without an irrelevant premise. Contrary to Rips' claim (1994), we found that overall one-model problems were easier than multiple-model problems. Moreover, one-model problems with no irrelevant premise were significantly easier than the multiple-model problems with no irrelevant premise in the case of the spatial problems. In the case of the temporal problems, this difference was almost significant ($P < 0.06$). This difference in particular is problematic for the rule-based theories. Indeed, Rips (1994) hypothesized that the search for a correct deduction could be sidetracked by the presence of irrelevant information in the multiple-model problems of Byrne and Johnson-Laird (1989); however, we observed a reliable difference between the one-model problems and the multiple-model problems when no irrelevant premise existed.

The results do not completely corroborate the model theory. One-model problems with an irrelevant premise were easier than the multiple-model problems with an irrelevant premise, but this difference was not reliable for the spatial problems or the temporal problems. Nevertheless, this lack of difference can be explained in terms of the model theory. If reasoners build only one of the possible multiple models, they can still arrive at the correct conclusion. Indeed, in the case of a multiple-model problem with a valid conclusion, both models yield the same valid conclusion. Moreover, we have evidence (see Schaeken & Johnson-Laird 1995) that with some practice reasoners can ignore an irrelevant premise in temporal problems such as problem 5. If reasoners use this strategy, then a multiple-model problem becomes a one-model problem. It is possible that the problems in the present experiment (which appear very difficult and contain many propositions) spontaneously elicited such a strategy.

There is, however, another plausible explanation for the similarity between one-model and multiple-model problems with an irrelevant premise. Reasoners could have eliminated one of the two models which were consistent with the first premise. The premise order might have focused participants on one of these models. Consider the first two premises of a multiple-model problem with an irrelevant premise:

The square is on the same card as the triangle.
The triangle is on the left of C.

On the basis of these premises, one can construct two models:

[□ | △] [C] [△ | □] [C]

It is possible, however, that the second model will be eliminated, given that the second premise seems to suggest: "The triangle is *immediately* on the left of C." Of course, reasoners can only decide to dispense with the second model after reading the following two premises of the multiple-model problem. These can be easily integrated in the remaining model:

* is in the same cell as the triangle.
$ is in the same cell as C.
What is the relation between * and $?

[□ | △*] [C$]

This would lead to the valid conclusion that * is to the left of $. Indeed, reasoners can come to this conclusion without considering the position of the square by simply focusing on the dyad △–C.

In contrast, reasoners are less likely to solve multiple-model problems with no irrelevant premise by means of the same strategy. The initial two premises are the same. Hence they are consistent with two models, although there may be a focus on the first model:

[□ | △] [C] [△ | □] [C]

However, given the third premise:

* is in the same cell as the square

it is impossible for the reasoners to ignore the position of the square. They must then consider the two possible models of the premises, which makes a multiple-model problem with no irrelevant premise more difficult than the multiple-model problem with an irrelevant premise. Somehow problematic, however, is the fact that reasoners had to solve only one spatial and one temporal problem, so it is not clear to us how this strategy should be elicited spontaneously.

The use of this simplification strategy was not possible for the participants in the experiments of Byrne and Johnson-Laird (1989). Consider the following multiple-model problem (problem 2):

B is on the right of A.
C is on the left of B.
D is in front of C.
E is in front of B.
What is the relation between D and E?

The second premise forces reasoners to construct the second model:

A C B C A B

They are obliged to keep these two models in mind in order to solve the problem. They cannot use the simplification strategy because of the contrast between "B is *immediately* on the right of A" and "C is *immediately* on the left of B" (the same is true for the participants in Schaeken et al. 1996 a, b):

A	C	B	C	A	B
	D	E	D		E

In conclusion, the present experiment provides additional support for the mental model theory. One-model problems are easier than multiple-model problems. Rips (1994) argued that the search for a correct deduction could be sidetracked by the presence of irrelevant information in the multiple-model problems, but we found that one-model problems with no irrelevant premise were easier than multiple-model problems with no irrelevant premise. Even the lack of difference between one-model and multiple-model problems with an irrelevant premise can be integrated in the model theory. Nevertheless, the mental model theory still lacks a clear description of the strategies used by reasoners.

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References

Braine, M. D. S., Reiser, B. J. & Rumain, B. (1984). Some empirical justification for a theory of natural propositional logic. *The psychology of learning and motivation* (vol. 18, pp. 313–371). New York: Academic Press.

Byrne, R. M. J. & Johnson-Laird, P. N. (1989). Spatial reasoning. *Journal of Memory and Language* **28**, 564–575.

Carreiras, C. & Santamaria, C. (1997). Reasoning about relations: spatial and nonspatial problems. *Thinking and Reasoning* **3**, 191–208.

Hagert, G. (1985). Modeling mental models: experiments in cognitive modeling of spatial reasoning. In: T. O'Shea (ed.), *Advances' in artificial intelligence* (pp. 179–188). Amsterdam: North-Holland.

Johnson-Laird, P. N. (1983). *Mental models: towards a cognitive science of language, inference and consciousness*. Cambridge, MA: Harvard University Press.

Johnson-Laird, P. N. & Byrne, R. M. J. (1991). *Deduction*. Hillsdale, NJ: Erlbaum.

Macnamara, J. (1986). *A border dispute: the place of logic in psychology.* Cambridge, MA: MIT Press.

Osherson, D. (1975). Logic and models of logical thinking. In: R. J. Falmagne (ed.), *Reasoning: representation and process in children and adults* (pp. 81–91). Hillsdale, NJ: Erlbaum.

Pollock, J. (1989). *How to build a person: a prolegomenon*. Cambridge, MA: MIT Press.

Rips, L. J. (1983). Cognitive processes in propositional reasoning. *Psychological Review* **90**, 38–71.

Rips, L. J. (1994). *The psychology of proof*. Cambridge, MA: MIT Press.

Schaeken, W. & Johnson-Laird, P. N. (1995). How do subjects reason about temporal relations. In: J. D. Moore & J. F. Lehman (eds.), *Proceedings of the Seventeenth Annual Conference of the Cognitive Science Society* (pp. 725–730). Mahwah, NJ: Erlbaum.

Schaeken, W., Johnson-Laird, P. N. & d'Ydewalle, G. (1996 a). Mental models and temporal reasoning. *Cognition* **60**, 205–234.

Schaeken, W., Johnson-Laird P. N. & d'Ydewalle, G. (1996 b). Tense, aspect, and temporal reasoning. *Thinking and Reasoning* **2**, 309–327.

Vandierendonck, A. & De Vooght, G. (1996). A comparison of reasoning with time and space concepts. *Thinking and Reasoning* **2**, 249–272.