Creative strategies in problem solving

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Abstract

Three experiments investigated how individuals solve "shape" problems. These problems are not susceptible to a means-ends strategy. They consist of a configuration of squares, whose sides consist of separate pieces; and the task is to remove a given number of pieces to leave behind a given number of squares. The paper presents a theory of how individuals develop strategies for these problems. Experiment 1 explored the constraints of symmetry and visual saliency in shape problems. Experiment 2 corroborated the theory's prediction of a major shift in which knowledge acquired during the evaluation of tactical steps comes to govern the generation of these steps. Experiment 3 showed that participants could be biased to adopt strategies making use of specific tactical steps.

Introduction

How do individuals develop strategies to solve problems? The question arises crucially for those problems that come in a series of different instances (e.g. Luchins's, 1942, water jug problems). Our aim was to answer this question for problems that do not have a unique solution and for which individuals cannot develop a simple deterministic strategy guaranteeing an error-free solution. We therefore studied what we refer to as "shape" problems (see Katona, 1940). Figure 1 presents an example of such a problem. There is an initial shape made out of separate pieces (matchsticks) and the goal is to remove a given number of pieces to leave a given number of squares. There are two constraints: the resulting squares should be of the same size as the initial squares, and the solution should not have any loose ends (pieces with an end not connected to any other piece).

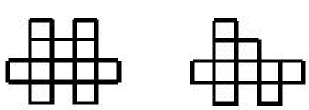
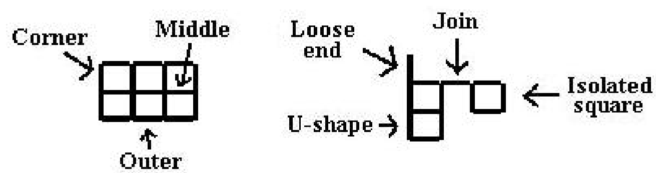


Figure 1: On the left is a shape problem in which the task is to remove five matches so that only ten squares remain. A solution is shown on the right.

An important feature of shape problems is that naïve individuals cannot use a means-ends strategy in which they work backwards from the desired goal (Newell & Simon, 1972). The goal merely specifies how many squares should remain, but not how they are arranged. Likewise, individuals cannot always tell if a tactical step in a shape problem makes progress towards the goal. The discovery of the tactical steps in shape problems is accordingly a discovery of the problem space. There are, in fact, seven distinct tactical steps for removing pieces, which are summarized in Figure 2.



1. To remove 1 piece & 0 squares, remove *loose end*
2. To remove 1 piece & 0 squares, remove *join*
3. To remove 1 piece & 1 square, remove *outer*
4. To remove 1 piece & 2 squares, remove *middle*
5. To remove 2 pieces & 1 square, remove *corner*
6. To remove 3 pieces & 1 square, remove *U-shape*
7. To remove 4 pieces & 1 square, remove *isolated-square*

Figure 2: The seven tactical steps for shape problems.

In what follows, we outline a theory of how individuals explore these tactical steps, and how they use these explorations to develop strategies. We then report three experiments that test the predictions of this theory.

The theory

Problem solving is a creative process, and we distinguish three main sorts of algorithm for creativity (e.g., Johnson-Laird, 1993). First, a *neo-Darwinian* algorithm consists of a stage in which ideas are generated followed by a stage in which they are evaluated. Generation depends on arbitrary combinations and modifications of existing elements; evaluation depends on the use of knowledge as constraints to filter out useless results. Any ideas that survive can be recycled recursively through the generative stage again, and so on. Second, in a *neo-Lamarckian* algorithm, all the knowledge acquired from experience constrains the generation of ideas. If alternatives are created, then choice amongst them can only be arbitrary, because all the constraints have already been used in their creation. When individuals have the requisite knowledge, the algorithm is highly efficient, because there is no need for recursion. Third, in a *multi-stage* algorithm, some knowledge is used to constrain the creation of ideas and some knowledge is used to evaluate the results – with the option of recursion. In sum, according to this account, constraints govern the evaluation of ideas, or their generation, or both.

The algorithm that individuals use to solve shape problems should depend on their experience. Naïve individuals are likely to tackle their initial problems using a strategy that is close to neo-Darwinian. They should be constrained solely by the statement of the problem, the problem shape itself, and their existing perceptual and cognitive processes. As they try out the various possible tactical steps, they learn their consequences, which are summarized in Figure 2. Learning occurs whether or not a tactical step turns out to be useful in solving a problem. Any problem allows only a limited set of tactical options, and so individuals should gradually narrow down the steps that are left to explore. Likewise, granted that the problem is within their competence, they should at length hit upon a sequence of steps that leads to a solution. In addition, some pieces in the problem shape are visually salient, and they may bias participants to attempt certain tactical steps first. Saliency is likely to depend on the perimeter of the shape. Any piece in the perimeter should be *visually salient* if it has at least one adjacent piece that is also in the perimeter and is at right angles to it. In addition, a piece should be more salient if both adjacent pieces are at right angles. A visually salient *component* comprises a group of such visually salient pieces that are adjacent to each other. These principles are probably special cases of broader factors governing visual salience. The acquisition of tactical knowledge depends on perceptual abilities, e.g., subitizing a small number of squares, and conceptual and inferential abilities, e.g., a grasp of the concepts of *squares* and *pieces*, and relevant arithmetical operations.

According to the theory, as tactical knowledge is acquired, it shifts from the evaluative stage of the creative process to the generative stage. Individuals accordingly shift from using a neo-Darwinian algorithm to a multi-stage algorithm, and may even converge on a neo-Lamarckian algorithm. This strategic shift enables them to avoid useless tactical steps and thereby to make more efficient progress towards solutions. They may proceed at once to correct tactical steps. A central component of an efficient strategy for shape problems is the *ratio* of the number of pieces to remove to the number of squares to remove (henceforth, the "p/s" ratio). It constrains the appropriate tactical steps from those afforded by the current configuration of the problem (see Figure 2). But, its optimal use depends on knowledge of the full variety of tactical steps. Conversely, a limited knowledge of these steps yields limited strategies for coping with the problems. Yet, the shift of tactical knowledge to the generative stage of problem solving should still occur, albeit with a restricted repertoire of tactical steps. Hence, it should be possible to bias the development of strategies by giving individuals only a limited experience of tactical steps in an initial set of problems.

Experiment 1

This experiment explored two factors that should affect shape problem solving: whether the initial shape is symmetrical or asymmetrical, and whether the solution is salient or not in the shape.

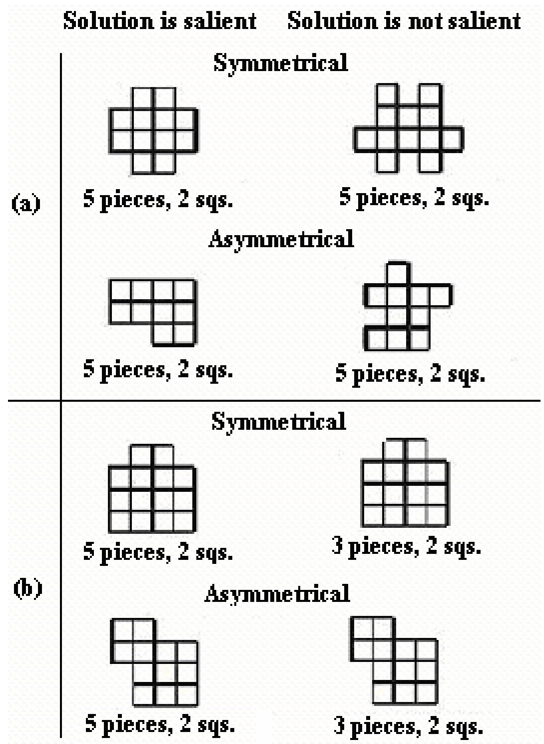


Figure 3: The eight problems used in Experiment 1. We manipulated symmetry and the presence of a salient solution.

Method and procedure

Twenty Princeton University students carried out eight problems, which manipulated symmetry and the presence or absence of salient solutions. Figure 3 presents the eight problems used in the experiment. In order to counterbalance the manipulation, one set of four problems (see Figure 3a) called for the same number of pieces and squares to be removed. This condition is feasible only by changing the shapes from the cases in which the solution is salient to the cases in which it is not. Hence, a second set of four problems (see Figure 3b) used the same shapes in these two cases but changed the number of pieces and squares that had to be removed. The experiment employed a block design: half of the participants carried out the four problems in Figure 3a first, and the other half carried out the four problems in Figure 3b first. The assignment of block presentation, as well as the order of problems within each block, was random.

On each trial, the participants constructed the shape in a given diagram using matchsticks. They then tried to solve the problem. They were told that they should not leave any loose ends, and that each square must consist of four pieces. They had to say "done" at the end of each trial to the experimenter, who recorded the latencies.

Results and discussion

Figure 4 presents the mean latencies to solve the eight problems. The two blocks did not differ reliably ($z = 1.61$, n.s.), and therefore we collapsed their latencies for analysis. Participants solved problems with a salient solution reliably faster than those without a salient solution (Wilcoxon signed-rank test, $z = 3.85$, $p<.001$). In addition, they also solved problems with a symmetric initial shape reliably faster than those with an asymmetrical initial shape (Wilcoxon signed-rank test, $z = 2.09$, $p<.05$). The two variables did not interact. These results demonstrate that existing factors in the problems can constrain problem solving strategies. To investigate how people develop strategies to cope with shape problems, however, participants would need to solve a series of problems calling for the removal of different numbers of pieces, and to think aloud as they solve the problems. Experiment 2 employed this procedure.

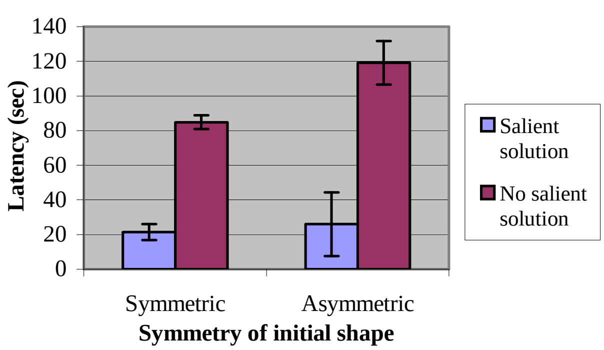


Figure 4: Experiment 1: Mean latencies as a function of symmetry and the presence of salient solution.

Experiment 2

This experiment tested the key prediction of a shift in strategy from a neo-Darwinian exploration of steps to their use in constraining the generation of steps. As a corollary, there should be a reduction in the number of steps that individuals take to solve problems.

Method and procedure

Fourteen Princeton University students carried out 12 problems presented in a random order. The problems (see Figure 5) varied in terms of symmetry, number of matches to be removed, and tactical steps, but they all called for removing two squares. The experimental procedure was the same as that in Experiment 1. However, there was an additional requirement: participants had to think aloud as they solved the problems. We video-recorded what they did and what they said.

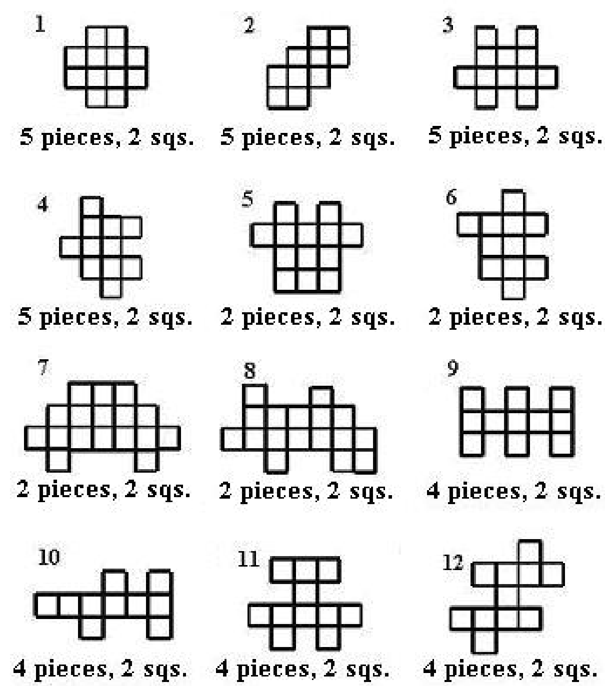


Figure 5: The 12 problem shapes used in Experiment 2.

Results and discussion

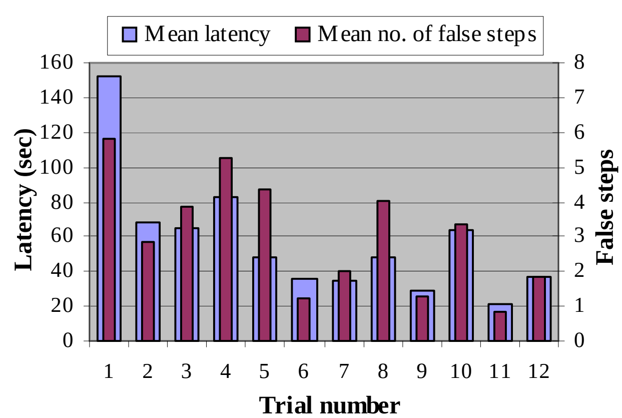


Figure 6: Experiment 2: Mean latencies and numbers of false steps across trials.

Figure 6 presents the mean latencies to solve the problems, and the mean numbers of false steps, over the 12 trials. A false step was one that the participants subsequently undid. As predicted, the participants were able, with experience, to solve the problems faster (Page's $L = 7882.5$, $z = 4.86$, $p <<$

.001), and to make fewer false steps (Page's L = 6750.0, z = 2.16, p <.05). These two variables correlated reliably for all but two of the problems (Pearson's r ranged from .65 to .95, with p <.05 to p <.001). In addition, in a post-experimental questionnaire, the participants were most likely to identify those tactical steps that they had used during the experiment: they all mentioned the *outer* and the *U-shape*, but none identified all seven tactical steps (see Figure 2).

The transcriptions of the video-recordings showed that the participants relied mainly on a single strategy, but there were other two strikingly different strategies. The main strategy has two stages. The first stage is exploratory: the participants try out various tactical steps, which they usually subsequently undo. They are acquiring knowledge of these steps, including steps irrelevant to the present problem. They are also acquiring knowledge of the *p/s ratio*, i.e., the ratio of pieces and squares to be removed (see the previous section). They grasped its relevance, but rarely in a complete way. The duration of this stage depends on the participants' experience with the problems. It accordingly shrinks in proportion over the problems as the participants acquire knowledge. The second stage of the strategy is the application of tactical knowledge. The participants consider the *p/s ratio*, often mentioning it explicitly, and use their tactical knowledge to select an appropriate tactical step. The shift has occurred from a neo-Darwinian strategy to a multi-stage strategy. Hence, the participants are able to make rapid progress to the solution. For some problems, they make no false steps. Likewise, they can combine tactical steps into a single step that solves the problem at a stroke. In other cases, they apply their knowledge recursively, removing a correct piece, re-assessing the number of pieces and the number of squares to be removed, and, as result, selecting a further tactical step, and so on, until they solve the problem. They are converging on a neo-Lamarckian strategy for solving shape problems.

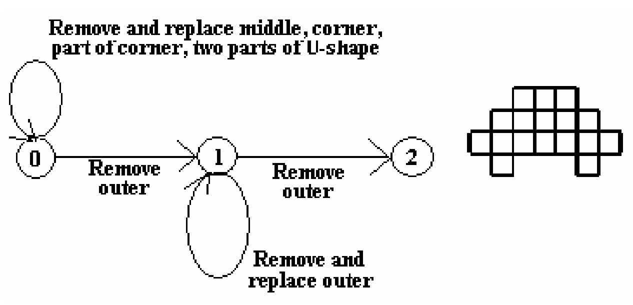


Figure 7: An example of the main strategy in a non-deterministic finite-state automaton. On the right, is the shape problem in which the goal is to remove 2 pieces and 2 squares (Problem 7).

Figure 7 shows an example of the main strategy. The participant started in an exploratory state (state 0) and tried various steps, which she immediately undid. She then correctly removed an *outer* (shifting to state 1), and then removed another outer to solve the problem. On some subsequent trials, e.g., with problem 5, she proceeded at once to the correct solution with no false steps. The protocol is typical in that it appears to reflect the use, not of a simple deterministic strategy, but one in which various steps are tried out in a way that appears to be non-deterministic.

One participant used a quite different sort of strategy. During the first stage of tackling a problem, the participant removed some pieces – often the required number – in an apparently arbitrary way, sometimes leaving several loose ends. The participant then carried out one of three actions: removing a new piece, replacing a piece removed earlier, or moving a piece from one position in the shape to fill the position of a piece that had been removed. The participant persisted in these steps until the solution emerged. The strategy was inefficient, yielding many more false steps than other participants. Yet, the participant gradually acquired some tactical knowledge, which became evident in both a more judicious initial removal of pieces and in more efficient steps in the second stage.

Another participant used a strategy that depended on the initial shape. The participant used the statement of the problem to divide the initial shape into two or three conceptual parts. For example, for Problem 2 (remove five pieces to remove two squares), the participant identified the number of squares to remain in the solution (eight), and then partitioned this number into two parts (three squares plus five squares). The participant then searched for ways to eliminate all but these configurations. Unfortunately, the attempt ignored the number of pieces to be removed. The strategy was inefficient, and yielded little tactical knowledge.

All three strategies stabilized as instances of the multi-stage algorithm outlined earlier. No-one developed a neo-Lamarckian strategy that guaranteed that they could proceed directly to the solution of a problem without any false steps.

Experiment 3

When individuals acquire a deterministic strategy, it transfers to new problems (see, e.g., Luchins, 1942). With shape problems, however, individuals do not acquire a deterministic strategy guaranteed to lead to solution, but instead acquire a tactical knowledge that constrains the generation of steps. Their resulting strategy does not appear to be deterministic (see Figure 7). Nevertheless, it should be possible to bias the development of strategies by giving participants an experience of only certain tactical steps in the initial problems. Experiment 3 tested this prediction.

The participants first encountered a series of four problems that could be solved only by using certain tactical steps. These tactics differed between two groups of participants. Both groups then tackled two "ambiguous" problems that could be solved using either set of tactics. A final unambiguous problem could be solved only with novel tactics, i.e., a problem used to train the participants in the other group. Such a problem should force the participants back to a greater use of the exploratory stage of their strategy.

Method

Twenty Princeton undergraduates were assigned at random to one of two groups: both carried out seven problems calling for the removal of four pieces to eliminate two squares. In Group 1, participants tackled four problems that could be solved only by removing two *corners*; in Group 2, they tackled four problems that could be solved only by removing a *U-shape* and an *outer*. Each participant carried out these trials in a different random order. Both groups then attempted two ambiguous problems, and finally a problem chosen randomly from the first four problems given to the other group. Figure 8 shows the complete set of problems. The experimental procedure was the same as that in Experiment 1.

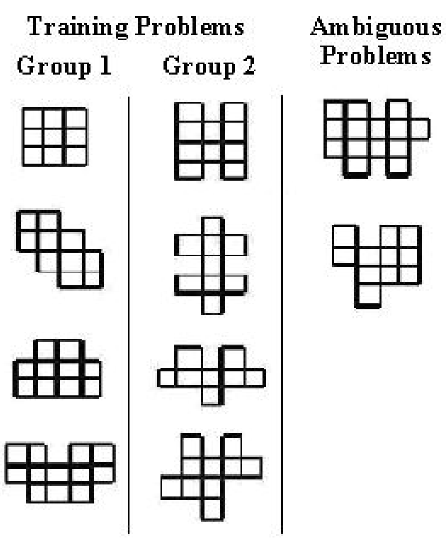


Figure 8: The problems used in Experiment 3. Each problem called for the removal of four pieces and two squares.

Results

Figure 9 presents the mean latencies of the two groups to solve the problems. The participants took progressively less time to solve the problems over the seven trials (Page's L = 1618.0, z = 4.23, $p << .001$). The ambiguous problems took slightly longer than the last training problem, though the difference was only marginally significant (Wilcoxon test, z = 1.64, $p > .05$). However, the choice of pieces to be removed showed that both groups persevered with the same tactics that they had used in training: overall, 92% of solutions were based on the same tactics; 14 out of the 20 participants used these tactics on both ambiguous problems, and the remaining participants were ties (Binomial test, $p << .001$). The final control problem took significantly longer to solve than the last training problem (Wilcoxon test, z = 2.61, $p < .01$).

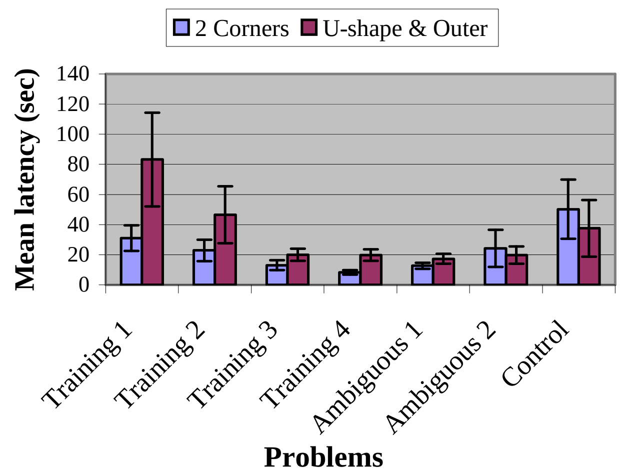


Figure 9: Mean latencies of the two groups in Experiment 3.

The results show that even when individuals have not acquired a deterministic strategy, their knowledge of tactics transfers to new problems. The training trials sufficed for the participants to develop tactical knowledge, and this knowledge constrained their search for solutions to the subsequent problems. With ambiguous problems, they readily succeeded though there was a marginal tendency for them to be slightly slower. In the case of the final problem, the tactics were inappropriate, and so they had to revert to a longer exploratory stage, which slowed them down.

General Discussion

Problem solving calls for creativity, because it calls for the generation of ideas that are novel (at least for the individual). In the case of, say, Duncker's X-ray problem (Duncker, 1945), psychologists can study only how individuals solve the problem for the first time. Hence, in order to investigate the development of strategies for solving problems, it is necessary to use problems that can be presented in a series that call for distinct solutions. In the past, such problems have been open to solution by a simple deterministic strategy of one sort or another (see, e.g., Luchins, 1942). In contrast, our goal was to examine the development of strategies for coping with problems that lie outside the bounds of a deterministic strategy – at least for our participants. We therefore studied problems that come in a series, just as many problems in daily life do – from the writing of computer programs to the search for a job.

Experiment 1 explored two constraints in shape problems, and found that symmetry of the initial problem shape, and the presence of salient solutions in the shape, facilitated problem solving. Experiment 2 showed that individuals do indeed normally begin to tackle such problems by exploring the consequences of various tactical steps in a way akin to a neo-Darwinian procedure. They choose a step arbitrarily, and then evaluate its consequences in relation to the solution of the problem. More importantly, however, they acquire knowledge of the number of squares that the step removes. They pick up this knowledge whether or not the step is useful in the solution of the problem. And, as the think-aloud protocols also showed, they acquire some understanding of the importance of the p/s ratio in determining appropriate steps for a given problem. This ratio, between the number of pieces to be removed and the number of squares to be removed, constrains the set of useful steps at any point in solving a shape problem. As the theory postulates, a strategic shift then occurs. Individuals start to use their knowledge of tactical steps and the ratio to govern the generation of tactical steps. In this way, they are able to avoid useless false steps in the solution of problems. No participant, however, was able to converge completely on a neo-Lamarckian strategy that guaranteed a solution to any problem without false steps. Indeed, it is an open question whether such a strategy is possible for shape problems of any degree of complexity.

Experiment 3 corroborated the prediction that constraints in the form of tactical knowledge do transfer to new problems. Participants acquired tactical knowledge during training trials, and they continued to use these tactics for problems that could be solved in other ways. When the tactics were inappropriate, they were slowed down because they had to revert to a longer exploratory stage to find the right tactics. Luchins (1942) discovered that deterministic strategies transfer in this way. Our results generalize his findings to show that even when experience leads at best to a strategy that is not deterministic, the strategy nevertheless transfers.

Is the strategic shift an instance of *insight*? The answer depends on what one takes insight to be (cf. Weisberg, 1986; Kaplan & Simon, 1990; Isaak & Just, 1995; Ormerod, MacGregor, & Chronicle, 2002). When the current constraints fail to yield a solution, the shift yields new constraints on the generation of tactical steps. This change, in turn, can yield the solution of a problem. The development of strategies for shape problems accordingly reflects a series of small insights in which constraints are changed as a result of strategic shifts.

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