THE PROBABILITY OF CONDITIONALS

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We report two studies investigating how naïve reasoners evaluate the probability that a conditional assertion is true, and the conditional probability that the consequent of the conditional is true given that the antecedent is true. The mental model theory predicts that individuals should evaluate the probability of a conditional on the basis of the mental models representing the conditional, and that evaluations calling for a greater number of models should be more difficult. It follows that the probability of a conditional should differ from the corresponding conditional probability. The results of the studies corroborated these predictions, and contrast with alternative accounts of naive evaluations of the probability of conditionals.

Key words: conditional, conditional probability, mental model, extensional reasoning

What is the probability that the following assertion is true?

A fair die thrown at random lands with one or six uppermost.

Naïve individuals, i.e., those who have not mastered the probability calculus, tend to answer correctly: 1/3. Now consider the probability of the following assertion:

There is a third World War in the Twenty first century.

Of course, it is not clear what would count as the correct answer, but nevertheless people are happy to assess the probability. The difference between the two sorts of inference reflects a distinction drawn by Tversky and Kahneman (e.g., 1983) between extensional and non-extensional judgments of probabilities. When individuals make an *extensional* judgment, they infer the probability of an event from the different possible ways in which it could occur. The probability calculus is accordingly a normative theory of extensional reasoning about probabilities. *Non-extensional* reasoning, as the pioneering studies of

Tversky and Kahneman (e.g.,1983) have shown, relies on some relevant heuristic, index, or evidence.

In extensional reasoning about probabilities, naïve reasoners appear to represent each possibility in a mental model, to assume that the possibilities are equiprobable unless they have evidence to the contrary, and to assess the probability of a proposition in terms of the subset of possibilities in which the proposition holds (Johnson-Laird, Legrenzi, Girotto, Legrenzi, & Caverni, 1999). According to this "subset" principle, they infer the probability of the disjunction about the dice by assessing the proportion of models in which the outcomes hold. The model theory predicts various phenomena, including the occurrence of systematic illusions. It also postulates that naïve individuals infer a conditional probability, p (B|A), not by using Bayes's rule, but by working out the subset of cases of A in which B holds (for experimental corroboration, see Girotto & Gonzalez, 2001, 2002; Johnson-Laird et al., 1999; see also Evans, Handley, Perham, Over, & Thompson, 2000; Sloman, Over, Slovak, & Stiebel, 2003). The theory also predicts that naïve individuals, including children, can solve probability problems asking for a simple combinatorial analysis of possibilities (for experimental evidence, see Girotto & Gonzalez, in press-a, -b). In certain predictable conditions, as these studies show, inferences about the chances of unique events are no harder than those about the frequencies of repeated events (pace Gigerenzer & Hoffrage, 1995; Cosmides & Tooby, 1996).

Extensional inferences of the probability of a disjunction, as in our opening example, are straightforward, but what about inferences of the probability of a conditional? Consider, for example, our first problem:

1) There are three cards face down on a table: an ace, a king, and a five. Paolo takes one card at random, and then Maria takes another at random.

   What is the probability that if Paolo has the ace then Maria has the king?

One influential view is that the probability of such a conditional, which has the form: If A then C, is close to the conditional probability of the consequent, C, given the antecedent, A, i.e., p (C|A). This view, which we refer to as the "conditional-probability" hypothesis, was formulated by Adams (1975), it has been defended by Stevenson and Over (1995, pp. 617–618), and has received some empirical support in recent studies (e.g., Evans, Handley, & Over, 2003; Hadjichristidis, Stevenson, Over, Sloman, Evans, & Feeney, 2001; Oberauer & Wilhelm, 2003).

One reason that individuals might conform to the conditional-probability hypothesis is that they may transform a problem in a subtle way, because the antecedent of a conditional is a subordinate clause. Given a subordinate clause, sentential operators, adverbials, and even questions that preface it, tend to be interpreted as though they apply only to the main clause. For instance, the conditional, "Is it true that if Paolo has the ace then Maria has the king?", is construed to mean, "If Paolo has the ace then is it true that Maria has the king?" Hence, a question of the form:

What is the probability that if A then C?

is readily re-interpreted as:

If A then what is the probability of C?

One way in which to try to block this re-interpretation is to formulate problems so that a

given individual, such as Vittorio, asserts the conditional, and the participants are then asked to assess the probability of Vittorio's assertion.

Evans and his colleagues have argued that conditionals should be treated as suppositional: they invite the listener to think about the hypothetical possibility in which their antecedents are true (e.g., Over & Evans, 2003). This idea is embodied in the well-known Ramsey test (a footnote in Ramsey, 1929/1990). Hence, given a conditional of the form, If A then C, individuals think about the possibility corresponding to its antecedent, A, and then they estimate their degree of confidence that the consequent, C, holds in that possibility. The result is a judgment of the probability of the conditional. As a corollary, Evans and his colleagues postulate that a conditional has a "defective" truth table with no truth value when its antecedent is false, i.e., it is a partial truth function. This idea can be traced back to Wason (1966), and thence to Quine (1952).

In what follows, our aim is to contrast the conditional-probability hypothesis with a theory based on mental models. We begin with this alternative theory and its account of the probability of a conditional. We then describe two experimental studies that examine the contrasting theories. Finally, we draw some general conclusions.

Mental Models and the Probability of Conditionals

Certain conditionals are *basic* in that they have a neutral content that is largely independent of context and background knowledge, and their antecedents and consequents have no semantic interrelations apart from their occurrence in the same conditional, e.g.:

If there is a circle then there is a triangle.

The model theory assumes that when individuals understand an assertion, they grasp the possibilities that it conveys. Typically, when they understand a basic conditional, they envisage the possibility described in the antecedent (they make a supposition as postulated in the Ramsey test). They interpret the consequent in relation to this supposition, e.g., if the consequent asks a question, they consider the answer in their model of the antecedent. If the consequent makes an assertion then they add its content to the model. The model theory, however, departs from a narrow interpretation of the Ramsey test. The theory allows that individuals realize that there are other possibilities compatible with the conditional: the antecedent is not necessarily true. They can accordingly envisage this possibility (Johnson-Laird & Byrne, 2002). They can construct a model representing the possibility in which the antecedent and consequent both hold, but they can also construct a model of the possibility in which the antecedent does not hold. This model is likely to be a "place holder", i.e., it has no content, because individuals do not normally consider the possibility explicitly. The two mental models are accordingly:

    ace   king

         ...

where "ace" represents the presence of an ace, "king" represents the presence of the king in the same possibility, and the ellipsis represents the implicit model.

When adults are asked to list all the possibilities compatible with a basic conditional, they can flesh out their mental models to turn them into *fully explicit* models (see, e.g., Barrouillet & Lecas, 1998). They accordingly enumerate the following possibilities for

the conditional:

    ace   king
    ¬ace   king
    ¬ace   ¬king

The core semantics of basic conditionals therefore yields possibilities corresponding to what logicians refer to as "material implication". But, the standard syntax and semantics of sentential logic makes no reference to possibilities. The meanings of clauses, referential relations between them, and general knowledge can all add temporal, spatial, and other relations to the interpretation of any sentential connective, including conditionals. This process of *modulation* can also prevent individuals from envisaging a possibility or it can help them to flesh out mental models into fully explicit models. Hence, the interpretative system is not a "truth functional" one, that is, it can never take for granted that the truth of a compound assertion depends merely on the truth values of the clauses that occur in the assertion (Johnson-Laird & Byrne, 2002).

When individuals have understood an assertion, i.e., considered the possibilities with which it is compatible, they may wonder whether the assertion is true or false. This ability, however, depends on the acquisition of the meta-linguistic notions, "true" and "false". These predicates apply to language itself depending on its relation to the situation under description. In logic, reference to truth and falsity occurs not in the language under analysis — the so-called *object* language, but in a *meta*-language, which is a language for talking about the object language. Natural language, however, is its own meta-language. Johnson-Laird (1990) argued that children first learn to use language to refer to possibilities, and only later acquire the ability to use meta-linguistic predicates. Similarly, adults have difficulty in assessing the truth or falsity of assertions containing connectives. Given a conjunction, for example, they often assume that its falsity implies that both of its clauses are false, and overlook that one clause can be true in a false conjunction (Byrne & Handley, 1992). When they judge the truth or falsity of a conditional, they tend to judge that it is true in a case in which both the antecedent and consequent hold, that it is false in a case in which the antecedent holds but the consequent does not, and that it is *irrelevant* to any case in which the antecedent does not hold (Johnson-Laird & Tagart, 1969; Evans, 1972). It was this phenomenon that led some theorists to defend the "defective" truth table in which conditionals have no truth value when their antecedents are false (e.g., Over & Evans, 2003). According to the model theory, however, the judgment of "irrelevance" is a direct consequence of the mental model with no explicit content. Individuals who have acquired the predicates "true" and "false" judge a conditional to be true if the situation corresponds to the explicit mental model, i.e., the antecedent and consequent both hold in the situation. They judge the conditional to be false if its antecedent is true, but its consequent is false. But, they consider the conditional irrelevant to any situation in which its antecedent does not hold, because such a situation corresponds to the model that has no explicit content. Hence, the model theory accounts for the two central phenomena of judgments about conditionals. On the one hand, naïve individuals list the three possibilities shown above when they are asked to list what is possible given a conditional. On the other hand, they make "defective" judgments of truth values.

Mental models are based on a principle of "truth". A model of a possibility represents clauses from the premises only when those clauses are true in the possibility, but not when they are false. Hence, an inclusive disjunction, such as:

There isn't an ace or there is king (and both propositions may be true)

has the following three mental models:

   ¬ace

        king

   ¬ace   king

where "¬" denotes negation. The first mental model does not represent the falsity of the clause that there is a king, and the second mental model does not represent the falsity of the clause that there isn't an ace, i.e., there *is* an ace. *Fully explicit* models, however, represent what is true and what is false:

   ¬ace   ¬king

    ace   king

   ¬ace   king

These possibilities are precisely those compatible with the basic conditional:

If there is an ace then there is king.

Hence, if individuals flesh out their models of basic conditionals explicitly, then they should be able to paraphrase disjunctions as conditionals, and vice versa. Ormerod and his colleagues have reported that their participants made such paraphrases (e.g., Richardson & Ormerod, 1997). We emphasize that the principle of truth applies by default. It does not imply that individuals never represent what is false or are unable to envisage the falsity of an assertion. These operations, however, are not the norm, and individuals make many more errors in listing possibilities that are false than in listing possibilities that are true (Barres & Johnson-Laird, 2003).

How do naïve individuals assess the extensional probability of a conditional? The answer depends on the *strategy* that they adopt. Individuals do indeed develop strategies to cope with reasoning problems (see, e.g., Van der Henst, Yang, & Johnson-Laird, 2002). In judgments of the probability of conditionals, the model theory predicts three potential strategies, which depend on how individuals assess the cases in which the conditional holds. We now explain the origins of these strategies.

Individuals may infer that the conditional is true only in the possibility corresponding to its explicit mental model (see Johnson-Laird et al., 1999, for corroboratory evidence). They then have two possible strategies for estimating the conditional's probability. In the "equiprobable" strategy, they discount the implicit model and so the only other prior possibility is the one in which the conditional is false. They accordingly infer that the probability of the conditional in problem 1 about Paolo is 1/2. In the "conjunctive" strategy, they construct the full partition of prior events but assume that the conditional is true only in the case corresponding to its one explicit mental model, i.e., they treat the conditional as analogous to a conjunction. They accordingly infer that the probability of the conditional in the Paolo problem is 1/6, because there are six a priori allocations of the cards. Finally, the use of fully explicit models of the conditional yields a "complete" strategy. They compute the complete partition and treat a conditional as true in any case

Table 1.   The Model Theory's Predictions for the Probabilities of Conditionals, Ordered According to Their Values From the Three Main Strategies, and for the Relevant Conditional Probabilities for the Three Sorts of Problems in Experiment 1

Task and strategy	Problem 1: Three cards: A, B, C. If Paolo has A, then Maria has C.	Problem 2: Three cards: A, B, C. If Paolo has A, then he has C.	Problem 3: Two cards: A, C. If Paolo has A, then Maria has C.
p (If A then C)			
Conjunctive	1/6	1/3	1/2
Equiprobable	1/2	1/2	1
Complete	5/6	2/3	1
p (C|A)	1/2	1/2	1

compatible with what is possible according to the conditional. They accordingly infer that the probability of the conditional in the Paolo problem is 5/6, because 5 out of the 6 possibilities are compatible with the conditional. Table 1 presents the consequences of the three strategies for three different problems, which we used in Experiment 1.

According to the model theory, individuals assess the conditional probability of C given A by considering the subset of cases of A in which C also holds (the subset principle). Hence, to assess the probability that Maria has the king given that Paolo has the ace, they consider the possibilities in which Paolo has the ace:

> *Paolo   Maria*
>   ace   king
>   ace   five

Only one of these possibilities satisfies the required condition (Paolo has an ace and Maria has a king), and so they should use the subset principle to infer correctly that the conditional probability equals 1/2. Readers should note that this correct estimate corresponds to the judgment of the probability of the conditional if individuals use the equiprobable strategy outlined earlier.

The analogous predictions can be made for a second problem:

2) There are three cards face down on a table: a 3, a 6 and an 8. Paolo takes one card at random, and then he takes another at random.

  Vittorio says:

   "If Paolo has the 8 then he also has the 3".

  What is the probability that Vittorio's assertion is true?

Reasoners who rely on mental models should construct the set:

> *Paolo*
>   8   3
>   ...

They should infer an *equiprobable* judgment of the probability of the conditional of 1/2. The fully explicit prior possibilities are:

Paolo

8   3

6   3

8   6

The *conjunctive* response is accordingly 1/3. Reasoners who rely on fully explicit models should grasp that the second possibility is compatible with the conditional, and so they should make the *complete* response of 2/3. Problem 2 should be easier than problem 1, because problem 2 yields fewer explicit possibilities: the two cards in Paolo's hand can be in either order, so only half the number of combinations is possible. To assess the conditional probability that Paolo has the 3 given that he has the 8, reasoners should envisage the two relevant possibilities:

Paolo

8   3

8   6

and infer correctly that the conditional probability is 1/2.

    A third problem is as follows:

    3) There are two cards face down on a table: a 7 and a 5. Paolo takes one card at random, and then Maria takes the other card. Vittorio says:

        "If Paolo has the 7, then Maria has the 5".

        What is the probability that Vittorio's assertion is true?

Reasoners who rely on mental models should construct the set:

Paolo	*Maria*
7	5
...	

But, there is no possibility for the antecedent of the conditional to be true and the consequent false. Hence, the *equiprobable* estimate of the probability of the conditional is 1. The fully explicit set of possibilities is:

Paolo	*Maria*
7	5
5	7

and so the *conjunctive* estimate is 1/2. Finally, those who grasp that both possibilities are compatible with the conditional should make the *complete* response that the probability of the conditional is 1. The conditional probability that Maria has the 5 given that Paolo has the 7 should be easy to infer: there is only one possibility compatible with Paolo having the 7, and in that possibility Maria has the 5, and so individuals should respond correctly that the conditional probability is 1.

    Table 1 summarizes the predicted values for these three problems. The model theory also embodies the principle that the greater the number of models that reasoners have to construct, the harder an inference is (see e.g., Johnson-Laird & Byrne, 1991, for corroboratory evidence). Four main predictions follow. First, the pattern of inferences should fit the values in Table 1, and so, contrary to the conditional-probability hypothesis, individuals should not invariably infer the same value for the probability of the conditional as for the conditional probability. Second, as a corollary, inferences about the

probability of a conditional should tend to underestimate its complete probability. Third, correct inferences about conditional probabilities should occur more frequently than complete inferences about the probabilities of conditionals. The former always depend on fewer models than the latter. Fourth, there should be the following increasing trend in the numbers of complete inferences about the probabilities of conditionals: problem 1 < problem 2 < problem 3. In contrast to the model theory, the conditional-probability hypothesis, which we outlined above, predicts that in all cases the probability of the conditionals should tend to correspond to the conditional probability of the consequent given the antecedent as a fact. The hypothesis makes no predictions about any differences among the three sorts of problem.

EXPERIMENT 1

Experiment 1 examined the contrasting predictions from the two hypotheses. The three sorts of problems, which we illustrated in the previous section (see Table 1), occurred on separate trials with different contents once with a question about the probability of a conditional and once with a question about a conditional probability.

Method

Participants: A total of 48 psychology undergraduates at Trieste University took part in the experiment as volunteers.

Materials and procedure: Each participant received a booklet containing the entire set of questions in a different random order, but with the constraint that the two questions based on problems of the same sort were not adjacent. Each of the problems was about a different set of cards. We used two different random assignments of specific cards to the problems, and we tested half of the participants with one assignment and the other half of the participants with the other assignment. The verbal formulation of each problem in the booklets is exactly as we stated in the earlier examples except that the actual problems were in Italian.

Results

Table 2 presents the numbers of participants making the various pairs of estimates for the probabilities of the conditionals and for the conditional probabilities. The category of "other" values in the Table combines idiosyncratic estimates made by no more than two participants, but when a greater number of participants made an unexpected response, the Table lists these responses as "unpredicted".

The results appear to corroborate the main predictions of the model theory. First, reasoners tended to make the responses that the model theory predicts, and, contrary to the conditional-probability hypothesis, they did not invariably make the same estimates for the probability of a conditional and the conditional probability. The conditional-probability hypothesis and the model theory make overlapping predictions, and so we examine their predictions separately for each of the three sorts of problems. This analysis puts the two accounts on an equal footing and yields equal *a priori* probabilities for their chance occurrence.

For problem1, the model theory predicts three possible values for the probability of the conditional (i.e., equiprobable, conjunctive, and complete values), and only the correct

Table 2. The Numbers of the 48 Participants in Experiment 1 Who Made the Stated Combinations of p (if A then C) and p (C|A) Evaluations for Each of the Three Sorts of Problem

Problem 1

Evaluation of p (if A then C)	Evaluation of p (C|A)				
	1/6	1/3	1/2*	Other values	Total
1/6 (Conjunctive)	0	1	7	1	9
1/3 (Unpredicted)	1	3	6	4	14
1/2 (Equiprobable)	0	5	10	1	16
5/6 (Complete)	0	0	0	0	0
Other values	0	2	4	3	9
Total	1	11	27	9	48

Problem 2

Evaluation of p (if A then C)	Evaluation of p (C|A)				
	1/3	1/2*	2/3	Other values	Total
1/4 (Unpredicted)	0	6	0	0	6
1/3 (Conjunctive)	5	8	2	1	16
1/2(Equiprobable)	1	8	1	0	10
2/3 (Complete)	1	2	1	0	4
Other values	3	5	0	4	12
Total	10	29	4	5	48

Problem 3

Evaluation of p (if A then C)	Evaluation of p (C|A)			
	1/2	1*	Other values	Total
1/2 (Conjunctive)	3	20	1	24
1 (Equiprobable and Complete)	0	19	1	20
Other values	0	3	1	4
Totals	3	42	3	48

Note: *=correct

value for the conditional probability. We compared the frequencies in these three cells with the other frequencies in the 3×3 cells in Table 2 (for Problem 1) corresponding to the rows for conjunctive, unpredicted, and equiprobable responses, and to the columns for 1/6, 1/3, and the correct response of 1/2 for the conditional probability. There was a small but reliable bias in favor of the model theory's predictions: 17 participants fit the prediction

and 16 participants did not fit the prediction (Binomial test with a prior of .33, $p<.025$). The conditional-probability hypothesis predicts that the probability of the conditional should equal the conditional probability. It also concerns three cells in Table 2 (for Problem 1): the three cases where both probabilities are equal to one another (with values of 1/6, 1/3, and 1/2). We compared the frequencies in these three cells with those in the 3×3 array of cells that contain them. There was no reliable bias in favor of the hypothesis: 13 participants fit the prediction and 20 participants did not fit the prediction (Binomial test with a prior of .33, *ns*). We note that this analysis favored the conditional probability hypothesis, because strictly speaking it predicts a single numerical value as the conditional probability (see Table 1), but we have tested merely whether the two probabilities (the probability of the conditional and the conditional probability) were the same. In fact, as Table 2 shows, a minority of participants produced the specific values predicted by the conditional-probability hypothesis (ten participants for problem 1, eight participants for problem 2, and 19 participants for problem 3).

For problem 2, the model theory again predicts the values in three cells (equiprobable, conjunctive, and complete values for the probability of the conditional paired with the correct value for the conditional probability). We compared the frequencies in these cells with the 3×3 array formed from the predicted values for the probability of the conditional and the three observed values for the conditional probability. The bias in favor of the prediction was reliable: 18 participants fit the prediction and 11 participants did not fit the prediction (Binomial test with a prior of .33, $p<.002$). The conditional-probability hypothesis also concerns the three cells in Table 2 (for Problem 2) in which both probabilities are equal to one another (with values of 1/3, 1/2, and 2/3). We compared the frequencies in these three cells with those in the 3×3 array of cells that contain them. There was no reliable bias in favor of the hypothesis: 14 participants fit the prediction and 15 participants did not fit the prediction (Binomial test with a prior of .33, *ns*).

For problem 3, the model theory predicts the values for two cells only, because the equiprobable and complete predictions are the same. We compared the frequencies in these cells with the 2×2 array formed from the predicted values for the probability of the conditional and the two observed values for the conditional probability. The bias in favor of the prediction was reliable: 39 participants fit the prediction and 3 participants did not fit the prediction (Binomial test with a prior of .5, $p<5$ in a billion). The conditional-probability hypothesis concerns the two cells in Table 2 (for Problem 3) in which both probabilities are equal to one another (with values of 1/2 and 1). We compared the frequencies in these two cells with those in the 2×2 array of cells that contain them. There was no reliable bias in favor of the hypothesis: 22 participants fit the prediction and 20 participants did not fit the prediction (Binomial test with a prior of .5, *ns*). We conclude that participants showed a reliable tendency to make judgments in accordance with the model theory rather than to infer that the probability of a conditional is the conditional probability of its consequent given its antecedent.

The second prediction of the model theory is that erroneous inferences about the probability of a conditional should tend to underestimate its complete probability. The results in Table 2 corroborate this prediction. In estimating the probability of the

conditional, 45 out of the 48 participants made more responses underestimating rather than overestimating its complete value, and no participants made more errors overestimating rather than underestimating its complete value (Binomial test, $z=6.56$, $p<.0003$).

The third prediction of the model theory is that inferences about conditional probabilities should be correct (68%) more often than the occurrence of complete inferences about the probabilities of conditionals (17% correct; Wilcoxon test, $z=11.8$, $p<.00005$).

The fourth prediction of the model theory is that there should be an increasing trend in the proportions of complete inferences about the probabilities of conditionals over the three problems. The participants made 0% complete responses to problem 1, 8% complete responses to problem 2, and 42% complete responses to problem 3. A priori, three problems yield eight patterns of possible responses, because each response to a problem is either a complete judgment or not. Of these eight patterns, two conform to the predicted trend, two go against it, and the rest are ties. In fact, the participants were biased in favor of the prediction and none yielded a pattern of responses against it (Binomial test, $p=20.5$). In general, the participants made the correct estimates of the conditional probabilities more often than chance (Binomial tests with a prior of .33, Problem 1, $z=4.6$, $p<.00003$; Problem 2, $z=10.9$, $p<.0003$; Problem 2, $z=5.5$, $p<.0003$).

Although the results corroborated the predictions of the model theory, they contained at least one unexpected result: for problem 1, 14 participants estimated the probability of the conditional as 1/3. The model theory, however, does provide a post hoc explanation of this judgment. The fully explicit models of the conditional (If Paolo has the ace then Maria has the king) are as follows:

Paolo	Maria
ace	king
¬ace	king
¬ace	¬king

Those individuals who judge that the conditional is true only in the case that both its antecedent and consequent are true, and who fail to move from the possibilities above to the full set of prior possibilities, should infer that the probability of the conditional is 1/3. In the case of problem 2, six participants unexpectedly judged the probability of the conditional as 1/4. Likewise, two of the conditional probability problems produced a reliable quantity of errors. In order to investigate possible causes, we carried out a replication of the experiment in which the participants had to think aloud as they tackled the problems.

A "Think Aloud" Replication

To throw some light on how individuals think about the problems, we tested a further 10 participants from the same population as before, and we instructed them to "think aloud" as they made their estimates for problems of the same sort that we used in our first experiment. The pattern of results was similar, but the unpredicted estimates of the probability of the conditional, which we observed in the previous experiment did not occur, i.e., values of 1/3 and a 1/4 for problems 1 and 2, respectively. Likewise, all the

estimates of the conditional probabilities were correct.

The "think aloud" protocols were revealing. They showed that one source of idiosyncratic responses was erroneous computations of the number of possible combinations, e.g., one participant computed the number of possibilities for problem 1 as 9, evidently merely squaring the number of cards. Some of the unpredicted responses in Experiment 1 may have been a result of an erroneous framing of the required computation.

Another major result was that despite our attempts to elicit judgments of the probability of the conditional *as a whole*, four of the participants assumed that the antecedent of the conditional was true, and accordingly made an estimate of the conditional probability of the consequent in this circumstances. Among these participants, only one reasoned in an explicitly hypothetical way in estimating the probability of the conditional in problem 1. He said: "Supposing that the premise 'Paolo has the ace' holds, Maria can have either the King or the 5. Hence, the probability that she has the King equals 50%, which is precisely the probability that the statement in this reformulation of the problem is true. Of course, the probability that Paolo has the ace and Maria has the king is different, i.e. 1/6". All the other participants took for granted that the antecedent of the conditional was true (e.g., "There are only two cards left on the table"), and produced their estimate on such a basis (e.g., "Hence, the probability that she has the King is 1/2").

We conclude that even couching requests for the probability of a conditional in terms of the question: "What is the probability that Vittorio's assertion is true?" fails to stop individuals from transforming the problem. They assume that Paolo has the ace, and then estimate the probability that Maria has the king. A defender of the conditional-probability hypothesis may say that such a procedure is precisely what the hypothesis predicts reasoners should follow. In our view, however, reasoners' protocols show that they are reformulating the problem as:

Given that Paolo has the ace, what is the probability Maria has the king?

It is this reformulation of the problem, by no means universal – as the results of our first experiment showed, rather than a general method of assessing the probability of conditionals that may give rise to estimates that seem to corroborate the conditional-probability hypothesis

EXPERIMENT 2

In Experiment 1 and its replication, the question about the probability of a conditional concerned a speaker's statement, whereas the question about the conditional probability did not, but instead asked directly for an estimate. Skeptics might argue that this difference contributed to the difference in the respective values that the participants created. Hence, Experiment 2 couched both questions in the same way, i.e., they both concerned the probability of a speaker's statement. The experiment used only problems 2 and 3, and the question about the conditional probability for problem 2 was as follows:

2) There are three cards face down on a table: a 3, a 6 and an 8. Paolo takes one card at random, and then he takes another at random. Paolo shows one of his cards: it

is the 8. Vittorio says "Paolo also has the 3". Given that, indeed, Paolo has the 8, what is the probability that this Vittorio's assertion is true?

Likewise, the question for problem 3 was as follows:

3) There are two cards face down on a table: a 7 and a 5. Paolo takes one card at random, and then Maria takes the other card. Paolo shows one of his cards: it is the 7. Vittorio says "Maria has the 5". Given that, indeed, Paolo has the 7, what is the probability that this Vittorio's assertion is true?

The three previous problems all called for the participants to construct multiple models, and so Experiment 2 included a problem that calls for only a single model:

4) There are two cards face down on a table: an ace and a five. Paolo takes both cards. Vittorio says:
    "If Paolo has the ace, then he also has the 5".
    What is the probability that Vittorio's assertion is true?

Individuals should readily construct the single model of the situation, and infer that the probability that Vittorio's assertion is true is 1. Likewise, they should readily be able to infer the conditional probability (of 1) couched in the following question:

Paolo shows one of his cards: it is the ace. Vittorio says "Paolo also has the 5". Given that, indeed, Paolo has the ace, what is the probability that this Vittorio's assertion is true?

Method

We tested 20 participants from the same population as before. The materials and procedure were the same as in Experiment 1, apart from the modifications indicated above.

Results

Table 3 presents the numbers of participants making the various pairs of responses for the probabilities of the conditionals and the conditional probabilities for problems 2 and 3. There were no unpredicted responses that were made by more than two participants. For problem 4, the model theory, as well as the conditional probability hypothesis, predict one possible value for both the probability of the conditional and the conditional probability (i.e., the correct value 1). Indeed, all participants produced this value. As in the previous studies, the results appear to corroborate the model theory's four predictions.

First, for problem 2, the model theory predicts three values (equiprobable, conjunctive, and complete) for the probability of the conditional, and the correct value for the conditional probability. As before, we compared the frequencies in these cells with the 3×3 array formed from the predicted values for the probability of the conditional and the three observed values for the conditional probability. There was a reliable bias in favor of the model theory's prediction: 12 participants fit the prediction and only one participant did not fit the prediction (Binomial test with a prior of .33, $p<.001$). The conditional-probability hypothesis concerns the three cells in Table 3 (for problem 2) in which both probabilities are equal to one another (with values of 1/3, 1/2, and 2/3). We compared the frequencies in these three cells with those in the 3×3 array of cells that contain them. There was no reliable bias in favor of the hypothesis: 6 participants fit the

Table 3.   The Numbers of the 20 Participants Making the Stated Combinations of p (if A then C) and p
           (C|A) Evaluations for Problems 2 and 3(Experiment 2)

Problem 2

Evaluation of p (if A then C)	Evaluation of p (C|A)				
	1/3	1/2*	2/3	Other values	Total
1/3 (Conjunctive)	0	4	0	1	5
1/2 (Equiprobable)	1	6	0	1	8
2/3 (Complete)	0	2	0	0	2
Other values	0	5	0	0	5
Total	1	17	0	2	20

Problem 3

Evaluation of p (if A then C)	Evaluation of p (C|A)			
	1/2	1*	Other	Total
1/2 (Conjunctive)	0	8	1	9
1 (Equiprobable-complete)	1	10	0	11
Total	1	18	1	20

Note: *=correct

prediction and 7 participants did not fit the prediction (Binomial test with a prior of .33, *ns*). For problem 3, the model theory predicts two values (conjunctive and equiprobable-complete) for the probability of the conditional, and the correct value for the conditional probability. We compared the frequencies in these cells with the 2×2 array formed from the predicted values for the probability of the conditional and the two observed values for the conditional probability. The bias in favor of the prediction was reliable: 18 participants fit the prediction and 1 participant did not fit the prediction (Binomial test with a prior of .5, *p*<.001). The conditional-probability hypothesis concerns the two cells in Table 3 (for problem 3) in which both probabilities are equal to one another (with values of 1/2 and 1). We compared the frequencies in these two cells with those in the 2×2 array of cells that contain them. There was no reliable bias in favor of the hypothesis: 10 participants fit the prediction and 9 participants did not fit the prediction (Binomial test with a prior of .5, *ns*).

Second, the results corroborated the prediction that erroneous judgments about the probability of a conditional tend to underestimate its complete probability. Indeed, 17 participants made more responses underestimating rather than overestimating its complete value, and only one participant made more errors overestimating rather than underestimating its complete value (Binomial test, *p*<.001).

Third, apart from Problem 4, which all participants responded to correctly,

inferences about conditional probabilities were correct (87%) more often than the occurrence of complete inferences about the probabilities of conditionals (32% correct). Sixteen participants were biased in favor of the prediction, and only one yielded a pattern of responses against it (Binomial test, $p<.002$).

Fourth, the results corroborated the prediction that there should be an increasing trend of complete inferences about the probabilities of conditionals over the three problems. The participants made 10% complete responses to problem 2, 55% complete responses to problem 3, and 100% complete responses to problem 4. Eighteen participants were biased in favor of the prediction and only one yielded a pattern of responses against it (Binomial test, $p<.002$).

In general, the participants made the correct estimates of the conditional probabilities more often than chance (Binomial tests with a prior of .33, $p<.001$ for all problems).

GENERAL DISCUSSION

The present results support the model theory of extensional reasoning about probabilities (Johnson-Laird et al., 1999). In order to infer the probability of a proposition, reasoners carry out three main steps: 1. they envisage the possibilities compatible with the premises; 2. they assume that these possibilities are equiprobable; and 3. they compute the probability of an assertion from the proportion of possibilities in which it holds. Suppose that, as in problem 1, Paolo and Maria each chose a card at random from the set: queen, king, ace, with no replacement, and that Vittorio asserts the conditional:

If Paolo has the ace then Maria has the king.

To infer the probability that this claim is true, individuals are likely to adopt one of three main strategies. They may assume that the conditional is true only in the possibility corresponding to its one explicit mental model, i.e., the possibility in which both the antecedent and the consequent are true. They may then consider that the only other possibility is the one in which the antecedent is true and the consequent is false. Hence, they infer that the conditional has a probability equal to the *equiprobable* value of 1/2. They may instead compute the probability of the conditional as the subset in which the antecedent and the consequent holds within *all* the prior possibilities given that each protagonist draws one card from the set of three (as shown in bold here):

Paolo	Maria
Ace	**King**
Ace	Queen
King	Ace
King	Queen
Queen	Ace
Queen	King

They accordingly infer the *conjunctive* probability of the conditional as 1/6. In both these cases, the participants are assuming that the conditional holds only when its one explicit mental model holds. But, if individuals do consider the fully explicit models of a

conditional, they can compute the probability of the conditional as the subset of all possibilities compatible with the conditional within the prior possibilities (as shown in bold):

Paolo	*Maria*
Ace	**King**
Ace	Queen
King	**Ace**
King	**Queen**
Queen	**Ace**
Queen	**King**

They accordingly infer the *complete* probability of the conditional as 5/6. This inference is difficult, in part because of the general principle that the more models that individuals have to consider, the harder the task should be. But, as our results showed, the computation does become easier with a smaller set of possibilities.

In general, naive reasoners' estimates of the probability of a conditional assertion differed from their estimates of the probability of the consequent given the antecedent. This finding contrasts with the conditional-probability hypothesis (Over, in press; Over & Evans, 2003; Stevenson & Over, 1995), and with three recent studies. Hadjichristidis et al. (2001) argued that their results corroborated the conditional-probability hypothesis. They also took the model theory to imply that reasoners construct only fully explicit models of the conditional. But, as we have seen, the model theory allows that individuals do *not* invariably construct fully explicit models (see Schroyens & Schaeken, 2004). Few participants in their second study inferred complete values, and so they were not constructing fully explicit models. Yet, only about half their participants produced estimates of the probability of the conditional corresponding to the appropriate conditional probability. Their other responses are, in fact, accounted for by the model theory. One mystery for both the theories to explain is the variety of estimates of conditional probabilities themselves.

Evans et al. (2003) also interpret their results as supporting the conditional-probability hypothesis. In one study, for example, the participants were given the description of a pack of 30 cards, containing yellow circles, yellow diamonds, red circles, red diamonds. They were asked:

> How likely is the following claim to be true of a card drawn at random from the pack: "If the card is yellow, it has a circle printed on it"?

Given a pack containing, say, 6 yellow circles, participants evaluated the conditional to be more likely if the pack contained only 2 yellow diamonds (mean rating of a probability of 54%) than if it contained 12 yellow diamonds (mean rating of a probability of 37%). Hence, the fewer counterexamples to the conditional in the deck, the greater was its rated probability. According to Evans et al., this result shows that the probability of a conditional is affected by the conditional probability of its consequent given its antecedent. They also take this result to rebut a material implication interpretation of the conditional, which they erroneously assume is the *only* interpretation posited by the model theory (see Schroyens & Schaeken, 2004, who point out this and other misinterpretations of the model theory). Individuals are certainly prepared to allow counterexamples to temper their

beliefs about the probability of conditionals. In other tasks, however, they take a counterexample to show that a conditional is false (Johnson-Laird & Byrne, 2002), and to assess its probability given the conditional to have a zero probability (Johnson-Laird et al., 1999).

Evans et al. did not test the conditional-probability hypothesis directly. Their participants did not judge both the probability of a conditional and the corresponding conditional probability. But, Oberauer and Wilhelm (2003) did elicit both judgments. Their participants had to imagine a deck of 2000 cards, with each card having an "A" or a "B" on it printed in either red or blue. They had to answer a conditional-probability question, such as:

One card was drawn at random from the deck, and it turned out to have an A. Estimate the probability that this letter is red.

Next, the participants were told that a random sample of 10 cards had been drawn from the deck, and they had to estimate the probability that the conditional, "If a card has an A on it, then it is red", was true for this sample of 10 cards. The experimenters interpreted the results as supporting the conditional-probability hypothesis. However, many participants produced estimates that did not support the hypothesis. The authors write: "... only a subset of participants understood the two [estimates] as equivalent. In Experiment 1B, 52% of participants gave the same value for p ($q|p$) and p (if p then q); only 23% of participants in Experiment 1A gave the same values consistently over all four conditions" (p. 685). We note that their experimental procedure called for direct estimates of the probability of conditionals, i.e., they did not introduce a speaker, such as Vittorio, who asserted the conditional. Their procedure is likely to increase the proportion of participants who translate the question about the probability of the conditional into a direct request for the conditional probability:

If A then what is the probability of C?

We note that in the recent studies it is likely to be difficult for naïve individuals to compute probabilities in an extensional way in those conditions in which they have to make judgments about a sample drawn from a population.

In contrast, our experiments called for estimates of both conditional probabilities and probabilities of conditionals; they made it relatively easy to think about the problems in an extensional way; and they tried to block the re-interpretation of the question about the probability of the conditional as a question about a conditional probability. Nevertheless, as the "think aloud" study showed, individuals still have a tendency to re-interpret the question as a request for an estimate of a conditional probability. It is likely to occur, because the antecedent of the conditional is a subordinate clause, and sentential operators are often taken to apply only to main clauses. Such methodological worries aside, there are more general arguments contrary to the conditional-probability hypothesis. We reported in the Introduction that with certain contents individuals paraphrase an inclusive disjunction of the form: Not-A or B, as a conditional of the form: If A then B, and vice versa. Yet, it is unlikely that individuals would assess the extensional probability of the disjunction as equal to p (B|A).

Our account has focused on extensional reasoning, i.e., on inferences about the

probability of a proposition based on the possible ways in which it might hold. Non-extensional reasoning, however, occurs when individuals cannot use the possibilities compatible with an assertion to estimate its probability. Such cases are typically those that concern unique events, e.g.,

If Bush is re-elected then peace in the middle East will be postponed.

Could the conditional probability hypothesis apply in estimating the probability of such conditionals? Once again, the difficulty in testing such a hypothesis is to ensure that participants do not re-interpret the question, transforming it into a direct request for a conditional probability.

When individuals rely on mental models, they tend to judge that a conditional is true only in the case that its antecedent and consequent hold. Their estimates of the probability of the conditional then depend on what other possibilities they infer. If they suppose that the only relevant alternative is one in which the antecedent holds and the consequent does not, then they infer an equiprobable estimate. If they consider all the alternative prior possibilities, but assume that the conditionals holds only when both the antecedent and consequent hold, then they infer a conjunctive estimate. Some individuals, however, do consider the fully explicit models of the conditional and of the prior possibilities, and use the relations between the two sets to infer a complete estimate. Finally, some individuals may estimate the probability of a conditional by computing the probability of the consequent given the antecedent. Perhaps they fall into the trap of assuming that the antecedent of the conditional is true, and then, given its truth, they compute the probability of the consequent. But, they may compute the equiprobable value for the probability of the conditional, which often, though not invariably, corresponds to the conditional probability (see Table 1). Our claim is therefore *not* that the conditional probability hypothesis is false, but rather that its use is merely *one* of the strategies that naïve individuals use in estimating the probability of a conditional extensionally. The set of strategies as a whole appear to support the theory of mental models.

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