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Mental models and cognitive change

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The theory of mental models owes its origins to Peirce's logic in the nineteenth century and to Craik's psychological research during the Second World War. This Special Issue marks the 30th anniversary of a book that tried to pull these and other strands together into a unified approach to comprehension and reasoning: *Mental Models*. The principal assumption of the theory is that individuals reason by trying to envisage the possibilities compatible with what they know or believe. The present paper reviews recent developments in the theory. It describes the issues that arise as a result of cognitive changes both in the short term and in the long term. And it introduces the set of papers in this Special Issue that explore such changes in mind.

Keywords: Cognitive change; Mental models; Review.

In the early 1970s, Arthur Fry was an engineer working for 3M, a vast manufacturing company. His colleague Spencer Silver had devised a new glue, but it was so weak that it could barely stick one piece of paper to another. It seemed useless. One Sunday, Fry was in the church where he sang in the choir. He marked the hymns to be sung in his hymnal with small pieces of paper, but much to his annoyance they kept falling out. During the sermon, he had an insight: Put Silver's glue on a piece of paper, and it would make a perfect bookmark. It would stick in place, but could be peeled off without damaging the page. Thus, the Post-it was born. Things looked one way to Mr. Fry at one moment, but in an instant his mind had changed utterly. Cognitive changes, including such insights, are the topic of this Special Issue of the *Journal of Cognitive Psychology*, and the Editors have asked me to outline the topic and to introduce the individual papers.

To understand cognitive change, the reader needs to understand certain aspects of thinking. Introspection yields merely one thought after another (if you're lucky). But, how does the mind generate such sequences? A helpful way to think about this problem is by analogy with a computer program. At any juncture in our waking lives, our minds have data, just as programs have data. Mental data can consist of an ephemeral perception or thought, but also long-term knowledge and beliefs. These data are often introspectible. But, our minds have ‘‘programs’’ that use these data, and these programs are inaccessible to consciousness. Yet, they must exist—in order to create new thoughts from old. Likewise, although we may be aware of data, we are not always aware of *how* our minds represent them. The questions of process and representation are psychological: The goal of cognitive psychology is to pin down their particular nature, precisely because of their

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inaccessibility. And this point raises the other half of the title of this Special Issue, namely, mental models.

THE THEORY OF MENTAL MODELS

No-one knows for certain the origins of the theory of mental models—henceforth, the *model* theory. The great American logician Charles Sanders Peirce had glimmers of such a theory in his diagrammatic accounts of reasoning, which, he wrote, "put before us moving pictures of thought" (Peirce, 1931–1958, vol. 4, para. 8). In fact, several nineteenth-century physicists, including Kelvin, Boltzmann, and Maxwell, had anticipated the idea of mental models (see Johnson-Laird, 2004). They too may have been anticipated by earlier thinkers. In the twentieth century, the great psychologist and physiologist Kenneth Craik (1943, Chap. 5) wrote:

> If the organism carries a "small-scale model" of external reality and of its own possible actions within its head, it is able to try out various alternatives, conclude which is the best of them, react to future situations before they arise, utilize the knowledge of past events in dealing with the present and the future, and in every way to react in a much fuller, safer, and more competent manner to the emergencies which face it.

Oddly, however, Craik believed that reasoning depends on verbal rules rather than on mental models.

Intimations of the modern theory of mental models go back to the 1970s, and the present author attempted to pull them together 30 years ago (Johnson-Laird, 1983). The theory has, of course, developed since then, and it rests on three fundamental principles. First, each mental model represents what is common to a distinct set of possibilities. So, given an assertion, such as "It's raining or else it's snowing", you have two mental models to represent each of the two possibilities (on the assumption that both can't be true). Second, mental models are *iconic*, that is, their structure as far as possible corresponds to the structure of what they represent. So, an assertion such as, "All the artists are bakers", has a model representing the relation between the two sets of individuals. Third, mental models based on descriptions represent what is true at the expense of what is false. This principle of *truth* reduces the load that models place on working memory, but it can lead to predictable errors in reasoning.

When we reason, we aim for conclusions that are true, or at least probable, given the premises. But, we also aim for conclusions that are novel, parsimonious, and that maintain information (Johnson-Laird & Byrne, 1991). Hence, we search for a property or a relation that was not among the premises (Goodwin & Johnson-Laird, 2005, 2008). An inference is *valid* if its conclusion is true in every case in which its premises are true: Validity preserves truth (Jeffrey, 1981, p. 1). According to the model theory, we draw a conclusion of necessity, probability, or possibility, depending on whether a conclusion holds in all, most, or some of the models of the premises (Byrne, 2005; Johnson-Laird, 2006). Much of our reasoning in daily life is a kinematic simulation of the world, and we draw conclusions or make decisions based on these simulations (Hegarty, 2004). We use simulation in order to create informal algorithms for carrying our various tasks (Khemlani & Johnson-Laird, 2012).

Like many cognitive theories—of which the most famous is due to Kahneman (2011)—the model theory distinguishes between intuitions and deliberations. This idea was part of the nascent model theory (see, e.g., Johnson-Laird & Steedman, 1978; Johnson-Laird & Wason, 1977, p. 5). And it has been pursued independently by Jonathan Evans and his colleagues (see, e.g., Evans, 2008) and other students of "dual process" theories (e.g., Sloman, 1996; Stanovich, 1999). According to the model theory, however, intuitions are based on heuristics in a system that has no access to working memory, and that therefore cannot carry out recursive processes (Hopcroft & Ullman, 1979). Hence, the system can cope with only one model at a time (Johnson-Laird, 1983, Chap. 6; Verschueren, Schaeken, & d'Ydewalle, 2005; Khemlani & Johnson-Laird, 2013a). It can do no arithmetic, not even counting, and so intuitions about the probabilities of unique events flout the probability calculus in egregious ways (Khemlani, Lotstein, & Johnson-Laird, 2011, 2012a), even leading to negative probabilities, which—quantum mechanics, apart—are nonsensical in daily life. In contrast, deliberations have access to working memory and so they can carry out recursive processes, such as a search for alternative models, and the assignment of numbers to intuitive probabilities. In sum, the distinction between intuitions and deliberations is in computational power: intuitions are not recursive, but deliberations can be. Unlike other accounts, the model theory of the two systems has been implemented in a computer program, mReasoner, that yields heuristic conclusions, that uses models of the evidence to infer the probabilities of events, and that deliberates about them (see Khemlani et al., 2012a, 2012b; the source code is at http://mentalmodels.princeton.edu/programs/mreasoner/).

Some years ago, the present author canvassed 100 distinguished psychologists with a simple question: What's wrong with the model theory? Critics pointed out some phenomena that the theory seemed unable to explain, but they also revealed several misconceptions about the theory. The alleged counterexamples to the theory all concerned conditional assertions, such as:

If the letter is G then the number is 4.

This conditional is compatible with any possibility except the one in which the letter is G but the number isn't 4. Adults with larger working memories were more likely to judge cases in which the letter wasn't G as irrelevant to the conditional's truth than those with smaller working memories (Evans, Handley, Nielens, & Over, 2007). And these authors took this result to be contrary to the model theory, because the latter response is correct. A rebuttal, as Schroyens (2010) points out, is that individuals should not take a single instance as sufficient to establish the truth of general rules, such as the one earlier. Critics adduced another putative counterexample to the theory: Individuals tend to infer the probability of a conditional, *if A then B*, as equal to the conditional probability of *B* given *A* (see also, e.g., Evans, Over, & Handley, 2005). This result is robust, but some individuals do base their judgments on all three possibilities compatible with a conditional (Girotto & Johnson-Laird, 2004). Moreover, it is easy to interpret:

What is the probability of if A then B?

as asking for the conditional probability:

If A then what is the probability of B?

This interpretation could account for the judgment of a conditional probability.

One common misconception of the model theory is that it is against rules. As an anonymous critic in my informal survey wrote:

> The problem is that the manipulation and processing of these symbols has to be done by rules, which are hidden in the computer program for the models or have to be understand by us at the meta-level. This fact has been one reason why critics of mental model theory have charged it with being a kind of mental logic and not a true alternative to it . . .

In fact, the model theory has never outlawed rules—it depends on grammatical rules, semantic rules for the construction of intensions, rules for mapping them into models, and rules for formulating conclusions. What it does not use are *formal rules of inference* for deriving conclusions from the logical forms of premises, and indeed it makes no use of logical form (see Johnson-Laird, 2010). Logicians distinguish between proof theory and model theory, both of which depend on rules, and prove relations between them. It would be a mistake to claim that because model theory uses rules, it is not a true alternative to proof theory. The whole of this argument was published over 20 years ago (Johnson-Laird & Byrne, 1991, p. 212), but the misconception persists.

Proponents of an approach to reasoning based on the probability calculus argue that both the model theory and theories based on formal rules are shackled to logic as a normative framework (e.g., Evans, 2012). What these proponents object to is that such an account cannot accommodate subjective beliefs, which vary in strength in a way that cannot be handled in logic (see also Oaksford & Chater, 2007). However, Ramsey (1931/1990) treated the theory of probability as a branch of logic. Indeed, it can be axiomatised in logic or in set theory. From its inception, the model theory has treated quantified assertions, such as, "All the artists are bakers", as establishing relations between *sets*. This approach generalises to quantifiers such as: "More than half of the artists", that cannot be defined using the quantifiers of first-order logic (Barwise & Cooper, 1981). Hence, if the model theory is shackled to anything it is, not logic, but set theory (see Johnson-Laird, 1983, Chap. 15).

A corollary is that the model theory gives an account of how individuals reason about probabilities (see Johnson-Laird, Legrenzi, Girotto, Legrenzi, & Caverni, 1999) and even accounts for subjective probabilities. Its fundamental assumption is:

> Each model represents an equiprobable alternative unless individuals have knowledge or

> beliefs to the contrary, in which case they will assign different probabilities to different models. (Johnson-Laird et al., 1999, p. 68)

Unaccountably, recent theorists have dropped the ''unless'' qualification, and argued that the theory postulates that possibilities are invariably assigned equal probabilities (see Ali, Chater, & Oaksford, 2011; Evans, 2008). So, how does the theory account for the varying degrees of subjective beliefs? The answer depends on models of the evidence, which yield intuitive and non-numerical representation of probabilities that can be mapped into numerical probabilities (see Khemlani, Lotstein, & Johnson-Laird, 2011, 2012, for the details of the computer program that carries out the entire process). One of the advantages of the model theory is that it opens the door to a general explanation of all forms of thinking with propositional content. But, how does it account for cognitive change? This Special Issue is designed to provide some answers, and so let us turn to the questions.

COGNITIVE CHANGES

The cognitive changes that concern us may occur in a second—as in Arthur Fry's epiphany that led to Post-its, or in a time frame of minutes, hours, or days. The changes themselves may have lasting effects on mental processes, or merely yield a particular outcome. In what follows, I use an informal taxonomy to introduce the papers in this Special Issue.

Perhaps the key distinction concerns the agent or cause of cognitive change. We can distinguish between intrinsic causes that occur within the mind, such as an emotion, and extrinsic causes that occur in the environment. One notable intrinsic cause is an awareness of an impasse or inconsistency. The detection of an inconsistency is a wonderful trigger for cognitive change. Years ago, Wason (1964) showed that when individuals are led to contradict themselves, they learn to abandon a fallacious pattern of inference that led to contradiction in the first place. William James (1907, p. 59) famously described what happens when an incontrovertible fact collides with beliefs: ''[The new fact] preserves the older stock of truths with a minimum of modification, stretching them just enough to make them admit the novelty''. This doctrine is known as minimalism, and it has garnered support among various cognitive scientists (e.g., Harman, 1986). An alternative view is that an inconsistency triggers, not minimal modifications to beliefs, but instead efforts to account for the origins of the inconsistency (Johnson-Laird, Girotto, & Legrenzi, 2004). A corollary is that revisions may or may not be minimal, but tend to depend on the construction of an explanation. Khemlani and Johnson-Laird (this issue, 2013b) report evidence that explicit revisions of inconsistence scenarios are faster when individuals have a causal explanation in mind. When explanations are difficult to construct, individuals attempt to modify their beliefs at the outset. Knauff, Bucher, Krumnack, and Nejasmic (2013) investigated how they do so in coping with indeterminate descriptions of the spatial positions of objects. These spatial descriptions are open to several different interpretations. Knauff et al.'s experiments show that individuals tend to construct a preferred initial model, but they can revise this model in order to make an alternative interpretation of the premises. And this process of revision does follow the principle of minimal change.

A thought or a perception often elicits an emotion. From Plato onwards, emotions are supposed to lead to irrationality. The facts seem otherwise. If an emotion is not of an incapacitating intensity, then it leads the person experiencing it to reason about its cause. As a result, the person is liable to reason well (see, e.g., Blanchette, Richards, Melnyk, & Lavda, 2007). Many psychological illnesses are disorders of emotion—the emotion is appropriate to the situation, but pathological in its intensity. It too can enhance reasoning about its cause comparison both to the patients' reasoning about other topics and to other individuals' reasoning about the same topic. Gangemi, Mancini, and Johnson-Laird (2013) report two experiments, which modify our understanding of the mechanism underlying the phenomena. The effect appears in the heuristic system for drawing intuitive conclusions rather than in the deliberative system for evaluating them: emotions in pathology evoke more conclusions, valid or invalid, about their causes, and fewer conclusions about other topics.

When individuals repeatedly carry out the same sort of reasoning, they spontaneously improve (Johnson-Laird & Steedman, 1978), and they even develop some smatterings of formal knowledge about the inferences they are making (Galotti, Baron, & Sabini, 1986). Similarly, when individuals carry out different instances of the same sort of problem, they learn the different tactical moves and their consequences. They don't always have to carry out these moves, because they can envisage their consequences. Moreover, as Lee and Johnson-Laird (2013) show, once they have this tactical knowledge, they can make a strategic shift in their method of solving the problems. Their knowledge of the consequences of different sorts of move comes to constrain their choice of actual moves. The authors describe both computational and experimental research corroborating this account.

One pertinent long-term development is an increase in the capacity of working memory during childhood. This increase enables various cognitive changes to occur. Barrouillet and his colleagues have charted how children develop in their ability to reason from conditional assertions. Their studies showed the processing capacity of working memory is more critical in predicting reasoning ability than chronological age (e.g., Barrouillet, Grosset, & Lecas, 2000). Children begin by treating conditionals as akin to conjunctions (one model), then they treat them as though they meant *if, and only if* (two models), and ultimately they home in on their correct conditional interpretation (three models). A long-standing mystery is the discrepancy between the three possibilities that individuals list as compatible with conditionals and their performance in evaluating the truth or falsity of conditionals in relation to states of affairs. When the if-clause of a conditional is false in a particular situation, they tend to judge the situation as possible given the truth of the conditional, yet as irrelevant to its truth or falsity in the truth-value task. One resolution treats the truth-value task as basic, and postulates a ''suppositional'' account of the evaluation of conditionals in which they have no truth value when their antecedents are false (see Evans et al., 2005). If the judgements of irrelevance are a consequence of the intuitive system of thinking, then they should be rapid and unaffected by a concurrent cognitive load. However, Vergauwe, Gauffroy, Morsanyi, Dagry, and Barrouillet (2013) argue that these judgements are a consequence of the deliberative system of thinking. Hence, they should be time consuming and reflect a process of building models. These authors report an experiment designed to determine which account is correct. Its results show that the judgements of irrelevance are the slowest of all in the truth-value task. This research contributes to the general picture of cognitive change in conditional reasoning.

Turning to the extrinsic causes of cognitive change, the most salient case is teaching. There is a long-standing tradition of efforts to improve deductive reasoning based on teaching participants logical principles (e.g., Cheng, Holyoak, Nisbett, & Oliver, 1986). An intensive form of training and analysis is due to Siegler and Svetina (2002), and it is known as ''microgenetics''. It calls for intensive testing over a period of days, and provides rich data about cognitive change. Ball (2013) used this technique to overcome a well-established bias in reasoning. Readers will recall from our earlier account that validity preserves truth. Consider this inference, for instance:

All of the musicians are English speakers.
Therefore, some of the English speakers are musicians.

It is valid, because if its premise is true then its conclusion is true. But, suppose that only some of the relevant musicians speak English: The premise is false, but the conclusion remains true. So, valid inferences sometimes lead from false premises to true conclusions. Naïve individuals have difficulty in divorcing truth and falsity from considerations of validity, and they are biased to accept invalid inferences if their conclusions are true, and to reject them if their conclusions are false. To correct the bias, Ball used the microgenetic procedure, giving feedback about correct responses during intensive training over several days. The results supported a dual-process account of belief bias based in part on assumptions about the search for alternative models. Nearly everyone benefited from the feedback during the microgenetic training, which reduced belief bias and enhanced logical performance. As the study showed, however, mere practice can also benefit logical responding, but without reducing the biasing influence of prior beliefs.

Schooling and the acquisition of literacy may affect reasoning. At one time, evidence suggested that peoples from subcultures with no writing or schooling were unable to make inferences about hypothetical matters (see, e.g., Luria, 1976; Scribner, 1977). But, an ingenious study showed that when an inference is framed in the context of a distant planet, about which no one could have any relevant knowledge, the bias disappears and the participants reason in a competent way about hypothetical individuals (Dias, Roazzi, & Harris, 2005). An analogous story can be told about the effects of culture on reasoning. At one time, evidence suggested that Westerners reasoned in an analytic and deductive ways, whereas Chinese, Japanese, and Koreans were intuitive, holistic, and tolerated contradictions (Nisbett, 2003). A reanalysis of these results has cast doubt on the difference (Unsworth & Medin, 2005). Likewise, some studies have been unable to detect any differences in reasoning between Chinese speakers in Hong Kong and Western students in Princeton (Lee & Johnson-Laird, 2006). Here is one trial from a task that was instructive:

> If a pilot falls from a plane without a parachute the pilot dies.
> This pilot didn't die.
> How come?

Participants can respond either with a deduction:

> The pilot didn't fall from a plane without a parachute.

or with an abduction based on their knowledge, such as:

> The plane was on the ground.

Most people make an abductive response to these problems. The bias occurred in the responses of the Chinese and the Americans, and the two groups did not differ reliably. Both groups, however, were more likely to make the deduction if they had just carried out a separate deductive experiment. Insofar as cultural differences in reasoning do exist, they seem to be in strategy rather than in underlying mechanisms of reasoning.

Does schooling change reasoning? Santamaría, Tse, Moreno-Ríos, and García-Madruga (2013) examined over a thousand children in secondary school in two grades (12 and 13 years of age). They found that the children's knowledge of the truth conditions of disjunctions and conditionals correlated with their deductive ability in a variety of tasks including syllogistic and conditional reasoning. They also used two metareasoning tasks: an evaluation of propositional attitudes and a modal syllogistic task. Their participants' performance depended on the number of models needed for an inference, and it improved from one grade to the next. Yet, the results—as the authors point out—could reflect a development in the processing capacity of working memory particularly for the metareasoning tasks.

Extrinsic factors can concern solely the way in which information is presented to individuals. A subtle effect is demonstrated in the research of Cutica and Bucciarelli (2013). When we talk, we often gesture. Why? One reason is to help us to keep track of where we are in a simulation (Hegarty, Mayer, Kriz, & Keehner, 2005). Another reason is that our gestures reflect the mental model that we are trying to convey (Núñez, 2006). Cutica and Bucciarelli propose that gestures can help individuals to form correct mental models of discourse with the corollary that they are then less likely to retain a verbatim memory for the discourse. As they showed, when their participants gestured whilst reading, the results corroborated these predictions.

An analogous effect can occur in framing a problem. As Murray and Byrne (2013) showed, if a problem depends on "insight" for its solution, such as Fry's problem of keeping his bookmarks in place, then framing can change the difficulty of the problem. Insight, in general, is a sudden grasp of a pertinent underlying idea without which the solution to a problem is impossible. The origins of insights are mysterious, but they certainly reflect a shift in cognition. Murray and Byrne argue that what foster solutions are factors that help to reduce the number of irrelevant models of possibilities. As their paper shows, these factors include redescriptions and hints that block common, but unhelpful, assumptions.

One final extrinsic factor that helps individuals to reason is a useful diagram. What counts as useful, however, is subtle. It calls for a diagram that helps individuals to envisage alternative possibilities. It then yields better reasoning than an equivalent verbal formulation. Other sorts of diagram, however, are no help at all (Bauer & Johnson-Laird, 1993). In a study of both spatial reasoning and reasoning in organic chemistry, Hegarty, Stieff, and Dixon (2013) focus on the cognitive change that occurs in the strategies that students use in reasoning from diagrams in organic chemistry. These strategies change as a consequence of taking a class in the discipline. At first, students primarily use simulation-based reasoning, in which they try to imagine the molecules as 3-D objects and mentally rotate them or imagine them from different perspectives. After the completion of the class, however, the use of simulation declines and tends to be replaced by diagrammatic manipulations and the use of rules. These changes, as the authors argue, depend on how the students are taught, and on their spatial abilities.

WHERE DO WE GO FROM HERE?

The reader, one hopes, goes on to read the papers in this issue. Their authors, one hopes, go on to resolve three great mysteries. The first mystery is the development of reasoning from infancy to adulthood. Piaget posed this problem (e.g., Inhelder & Piaget, 1958). But, his attempts to solve it erred in opposite ways, attributing too little ability to infants and too much to adults (for a review, see Chap. 18, Johnson-Laird, 2006). The second mystery is how to improve reasoning. Everyone makes mistakes—in part, because almost all domains of reasoning are computationally intractable so that, as the number of independent premises increases, it rapidly becomes impossible for any finite organism to cope (see Ragni, 2003, for a proof that even simple two-dimensional spatial reasoning is intractable). Yet, some preliminary efforts to teach people to enumerate possibilities (due to Victoria Bell in unpublished studies) show promise in improving reasoning. The third mystery is how to integrate all sorts of reasoning into a single framework. Leibniz dreamt of a mechanical system that would resolve all arguments, but logic now looks unlikely to provide such a system. Perhaps cognitive science can transform the dream into reality.

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